Section 9.6 Solving Exponential Equations and Logarithmic Equations

Base-Exponent Property

For any $a > 0,\ a \neq 1,$

$$a^x = a^y \longleftrightarrow x = y.$$

Example 1 Solve $2^{3x-7} = 32.$

$$2^{3x-7} = 32$$
$$2^{3x-7} = \boxed{}$$
$$3x - 7 = 5$$
$$3x = \boxed{}$$
$$x = \boxed{}$$

Check:

$$\begin{array}{c|c} 2^{3x-7} & 32 \\ \hline 2^{3\cdot\boxed{}-7} & ?\ 32 \\ 2^{\boxed{}} & \\ 32 & 32 \quad \text{TRUE} \end{array}$$

The solution is $\boxed{}$.

Property of Logarithmic Equality

For any $M > 0,\ N > 0,\ a > 0,$ and $a \neq 1,$

$$\log_a M = \log_a N \longleftrightarrow M = N.$$

Example 2 Solve $3^x = 20.$

$$3^x = 20$$
$$\log 3^x = \log 20$$
$$\boxed{}\log 3 = \log 20$$
$$x = \frac{\log 20}{\boxed{}}$$
$$x \approx 2.7268$$

Check: $3^{2.7268} \approx \boxed{}$

The solution is about $\boxed{}$.

Example 3 Solve $100e^{0.08t} = 2500$.

$$100e^{0.08t} = 2500$$
$$e^{0.08t} = \boxed{}$$
$$\ln e^{0.08t} = \ln 25$$
$$\boxed{} = \ln 25$$
$$t = \frac{\ln 25}{\boxed{}}$$
$$t \approx 40.2$$

The solution is about $\boxed{}$.

Example 4 Solve $4^{x+3} = 3^{-x}$.

$$4^{x+3} = 3^{-x}$$
$$\log 4^{x+3} = \log 3^{-x}$$
$$\left(\boxed{}\right)\log 4 = \boxed{}\log 3$$
$$x\log 4 + \boxed{}\log 4 = -x\log 3$$
$$x\log 4 + \boxed{} = -3\log 4$$
$$\boxed{}\left(\log 4 + \log 3\right) = -3\log 4$$
$$x = \frac{-3\log 4}{\boxed{}}$$
$$x \approx -1.6737$$

The solution is about $\boxed{}$.

Example 5 Solve $e^x + e^{-x} - 6 = 0$.

$$e^x + e^{-x} - 6 = 0$$

$$e^x + \frac{1}{\boxed{}} - 6 = 0$$

$$\boxed{}\left(e^x + \frac{1}{e^x} - 6\right) = \boxed{} \cdot 0$$

$$e^{2x} + 1 - 6e^x = 0$$

$$e^{2x} - 6e^x + 1 = 0 \qquad e^{2x} = \left(e^x\right)^2$$

$$\left(\boxed{}\right)^2 - 6e^x + 1 = 0$$

$$u^2 - 6 \cdot \boxed{} + 1 = 0 \qquad \text{Let } u = e^x.$$

$a = 1 \quad b = -6 \quad c = 1$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\left(\boxed{}\right) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{6 \pm \sqrt{\boxed{}}}{2} = \frac{6 \pm \boxed{} \cdot \sqrt{2}}{2} = 3 \pm \boxed{}$$

$$e^x = 3 \pm 2\sqrt{2}$$

$$\ln e^x = \boxed{}\left(3 \pm 2\sqrt{2}\right)$$

$$\boxed{} \cdot \ln e = \ln\left(3 \pm 2\sqrt{2}\right)$$

$$x = \ln\left(3 \pm 2\sqrt{2}\right)$$

The solutions are approximately $\boxed{}$ and $\boxed{}$.

Example 6 Solve $\log_3 x = -2$.

$$\log_3 x = -2$$
$$3^{\square} = x$$
$$\frac{1}{3^2} = x$$
$$\boxed{} = x$$

Check:

$$\begin{array}{c|l} \multicolumn{2}{c}{\log_3 x = -2} \\ \hline \log_3 \boxed{} & ?\ -2 \\ \log_3 \frac{1}{3^2} & \\ \log_3 3^{-2} & \\ \boxed{} & -2 \end{array}$$

____
TRUE / FALSE

The solution is $\frac{1}{9}$.

Example 7 Solve $\log x + \log(x+3) = 1$.

$$\log x + \log(x+3) = 1$$
$$\log_{10}\left[x\left(\boxed{}\right)\right] = 1$$
$$x(x+3) = \boxed{}^{1}$$
$$x^2 + \boxed{} = 10$$
$$x^2 + 3x - 10 = 0$$
$$(x-2)\left(\boxed{}\right) = 0$$
$$x - 2 = 0 \quad \textit{or} \quad x + 5 = 0$$
$$x = \boxed{} \quad \textit{or} \quad x = \boxed{}$$

Check: For 2:

$$\begin{array}{r|l} \multicolumn{2}{c}{\log x + \log(x+3) = 1} \\ \hline \log \boxed{} + \log\left(\boxed{} + 3\right) & ?\ 1 \\ \log 2 + \log 5 & \\ \log(2 \cdot 5) & \\ \log 10 & \\ 1 & 1 \quad \text{TRUE} \end{array}$$

Check: For -5:

$$\begin{array}{r|l} \multicolumn{2}{c}{\log x + \log(x+3) = 1} \\ \hline \log\left(\boxed{}\right) + \log(-5+3) & ?\ 1 \end{array}$$

We see that -5 ____________ be a
can / cannot
solution because the logarithmic function is not defined for negative values of x.

The solution is $\boxed{}$.

Example 8 Solve $\log_3(2x-1)-\log_3(x-4)=2$.

$$\log_3(2x-1)-\log_3(x-4)=2$$
$$\log_3\frac{2x-1}{\boxed{}}=2$$
$$\frac{2x-1}{x-4}=\boxed{}^2$$
$$\frac{2x-1}{x-4}=9$$
$$(x-4)\cdot\frac{2x-1}{x-4}=(x-4)\cdot 9$$
$$2x-1=9x-36$$
$$35=7x$$
$$5=x$$

Check:

$$\begin{array}{r|l} \log_3(2x-1)-\log_3(x-4) & 2 \\ \hline \log_3(2\cdot 5-1)-\log_3(5-4) & ?\ 2 \\ \log_3\boxed{}-\log_3\boxed{} & \\ 2-\boxed{} & \\ 2 & 2 \quad \text{TRUE} \end{array}$$

The solution is $\boxed{}$.

Example 9 Solve $\ln(4x+6)-\ln(x+5)=\ln x$.

$$\ln(4x+6)-\ln(x+5)=\ln x$$
$$\ln\frac{\boxed{}}{x+5}=\ln x$$
$$\frac{4x+6}{x+5}=\boxed{}$$
$$(x+5)\cdot\frac{4x+6}{x+5}=(x+5)\cdot x$$
$$4x+6=\boxed{}+5x$$
$$0=x^2+x-6$$
$$0=(x+3)\left(\boxed{}\right)$$

$x+3=0$ *or* $x-2=0$

$x=\boxed{}$ *or* $x=\boxed{}$

The number -3 ________ a solution because $4(-3)+6=-6$ and $\ln(-6)$ is not a real
is / is not

number. The value $\boxed{}$ checks and is the solution.