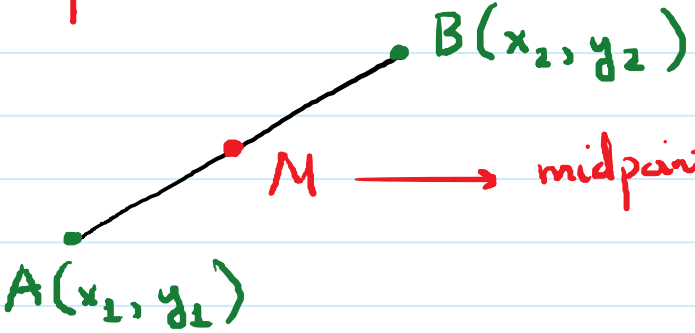
Distance and Midpoint Formulas and Circles

Monday, March 4, 2019 1:03 PM

Midpoint Formula:

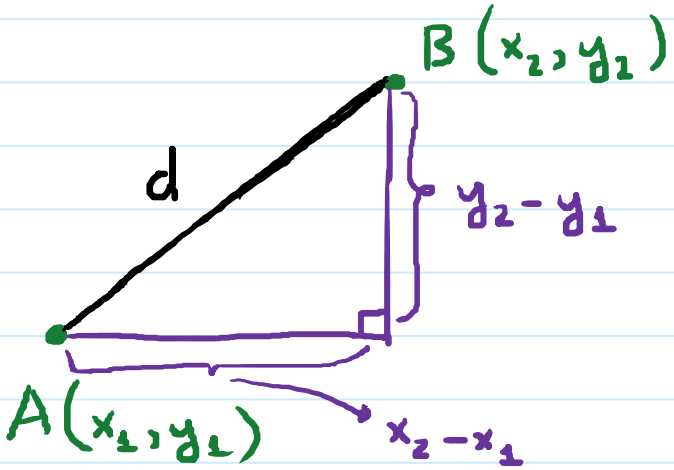


M ⟶ midpoint of line segment AB

$$x_M = \frac{x_1 + x_2}{2} \; ; \; y_M = \frac{y_1 + y_2}{2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Distance Formula:



d = distance from A to B

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Distance from A to B} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

E.g. Find the midpoint and distance between 2 given points.

(a) $(4,6)$ and $(-5,-6)$ (b) $(5,0)$ and $(1,3)$

Sol:

(a) Midpoint $M = \left(\frac{4+(-5)}{2}, \frac{6+(-6)}{2}\right)$

$$\boxed{M\left(-\frac{1}{2}, 0\right)}$$

$$d = \sqrt{(-5-4)^2 + (-6-6)^2}$$

$$= \sqrt{81+144} = \sqrt{225} = \boxed{15}$$

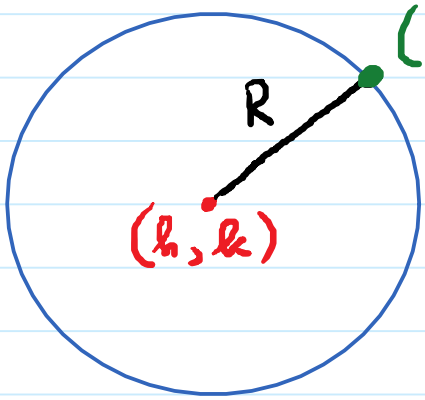
(b) Midpoint $M = \left(\frac{5+1}{2}, \frac{0+3}{2}\right)$

$$M\left(3, \frac{3}{2}\right)$$

$$d = \sqrt{(1-5)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = \boxed{5}$$

(3) Circle.

A circle is the collection of points that have the same distance to a given point.



center = (h,k)

Radius = R.

Distance from (x,y) on circle to center (h,k) equals R.

$$\sqrt{(x-h)^2+(y-k)^2} = R$$

Square both sides:

$$\boxed{(x-h)^2+(y-k)^2 = R^2}$$

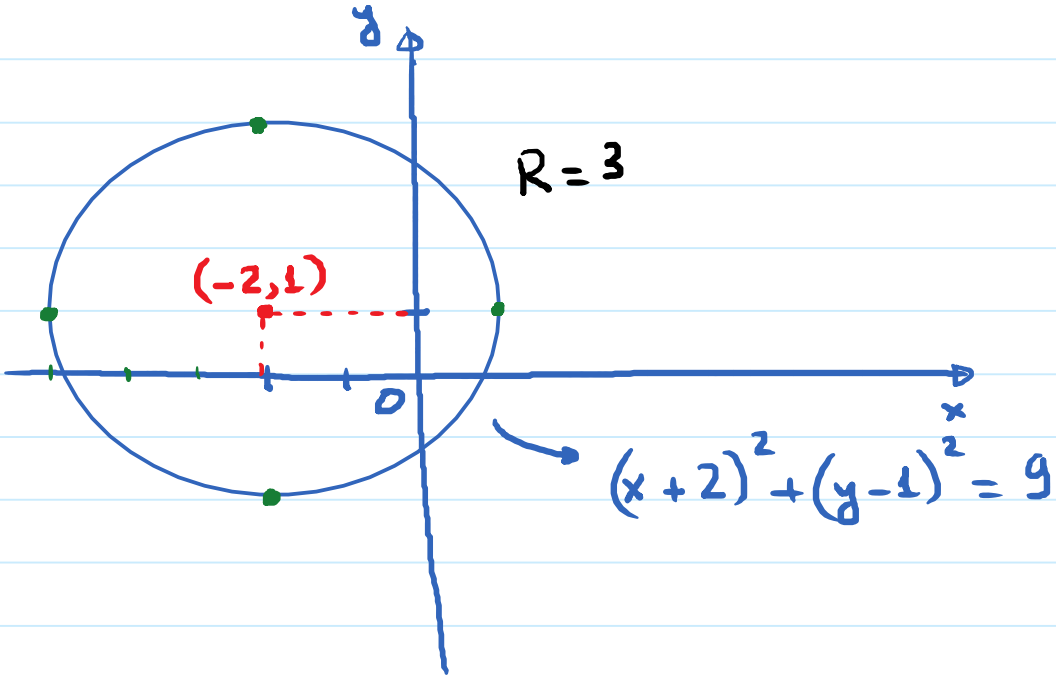
This is the equation of the circle with center = (h,k) and radius = R.

E.g. Find the equation of the circle with center = $(\underset{h}{-2}, \underset{k}{1})$ and radius = $\underset{R}{3}$.

Sol: Equation:

$$(x-(-2))^2+(y-1)^2 = (3)^2$$

$$\boxed{(x+2)^2+(y-1)^2 = 9}$$



E.g (a) Find the equation of the circle with center = $(-1,1)$ and passes through $(2,5)$

(b) Find the equation of the circle given that the points $(-8,1)$ and $(2,7)$ are the endpoints of a diameter.

Sol:

(a) $(x-(-1))^2+(y-1)^2=R^2$

$(x+1)^2+(y-1)^2=R^2$

Since $(2,5)$ belongs to circle; we have

$(2+1)^2+(5-1)^2=R^2 \rightarrow R^2=25 \rightarrow R=5$

Equation: $\boxed{(x+1)^2 + (y-1)^2 = 25}$

(b) Center = Midpoint of a diameter.

$$\text{Center} = \left(\frac{-8+2}{2}, \frac{1+7}{2}\right)$$

$$\text{Center} = (-3, 4)$$

$$\rightarrow \text{Equation: } (x-(-3))^2 + (y-4)^2 = R^2$$

$$(x+3)^2 + (y-4)^2 = R^2$$

Since $(2,7)$ is on circle, we have:

$$(2+3)^2 + (7-4)^2 = R^2$$

$$25 + 9 = R^2 \rightarrow R^2 = 34 \rightarrow R = \sqrt{34}$$

Equation: $\boxed{(x+3)^2 + (y-4)^2 = 34}$

The general form of the equation of a circle.

The form $(x-h)^2 + (y-k)^2 = R^2$ is called the Standard Form of the circle.

E.g. of an equation in general form:

$$x^2 + y^2 + 2x + 8y + 6 = 0.$$

This is the general form of the equation of a circle.

Q: How do we find center and radius from this form?

A: We need to convert it to Standard Form.

→ Completing the square:

$$\underbrace{x^2 + 2x + 1}_{} + \underbrace{y^2 + 8y + 16}_{} = -6 + 1 + 16$$

$$(x+1)^2 + (y+4)^2 = 11$$

→ Center: $(-1, -4)$; Radius: $\sqrt{11}$.

Ex. Convert to Standard form and find center and radius:

(a) $x^2 + y^2 + 10x + 20 = 0$ → Center: $(-5, 0)$, Radius: $\sqrt{5}$

(b) $x^2 + y^2 - 4x + 6y + 10 = 0$ → C: $(2, -3)$, R: $\sqrt{3}$

(a) $x^2 + y^2 + 10x + 20 = 0$

$x^2 + 10x + 25 + y^2 = -20 + 25$

$(x+5)^2 + y^2 = 5$

→ Center: $\boxed{(-5, 0)}$; $\boxed{R = \sqrt{5}}$

(b) $x^2 + y^2 - 4x + 6y + 10 = 0$

$x^2 - 4x + 4 + y^2 + 6y + 9 = -10 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 3$

Center: $\boxed{(2, -3)}$; $\boxed{R = \sqrt{3}}$