Domain of a Function.

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I) Rational Function:

$$f(x) = \frac{p(x)}{q(x)}$$

To find domain: Set denominator $q(x) = 0$.

Then Solve for x. Domain = all real #s

except for the values you just solved for.

<u>Note:</u> If the equation $q(x) = 0$ has no real

solutions, then Domain = all real #s.

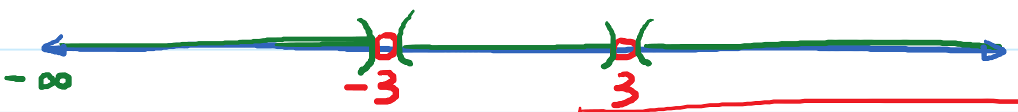
<u>E.g.</u> Find the domain of the given function:

(a) $f(x) = \dfrac{x}{9 - x^2}$ (b) $g(x) = \dfrac{x+8}{8x^3 - 2x^2 - 3x}$

(c) $h(x) = \dfrac{x}{4x^2 + 12}$

<u>Sol:</u> (a) Set $9 - x^2 = 0 \rightarrow -x^2 = -9$

$\rightarrow x^2 = 9 \rightarrow x = \pm 3$



Domain in interval notation: $\boxed{(-\infty, -3) \cup (-3, 3) \cup (3, \infty)}$

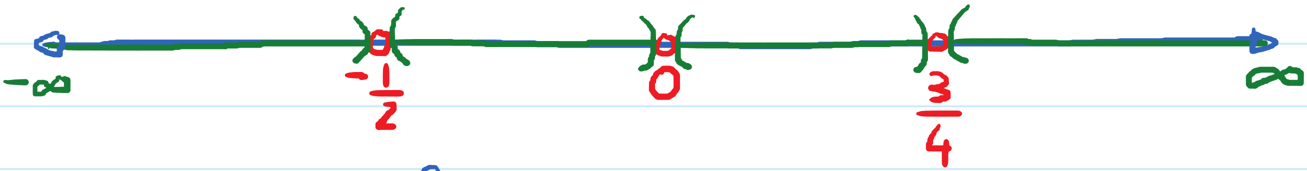
(b) Set $8x^3 - 2x^2 - 3x = 0$

$$x(8x^2 - 2x - 3) = 0$$

$$x(4x-3)(2x+1) = 0$$

$$x = 0 \ ; \ 4x - 3 = 0 \ ; \ 2x + 1 = 0$$

$$x = \frac{3}{4} \ ; \ x = -\frac{1}{2}$$



Domain in interval notation:

$$\boxed{\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)}$$

(c) Set $4x^2 + 12 = 0 \longrightarrow 4(x^2 + 3) = 0$

$$\rightarrow x^2 + 3 = 0 \rightarrow x^2 = -3 \rightarrow x = \pm\sqrt{-3}$$

$\rightarrow x = \pm i\sqrt{3} \longrightarrow$ Non-real solutions.

Domain = all real #s = $\boxed{(-\infty, \infty)}$

II Domain of a function with radical.

To find the domain of a function of the form

$$\sqrt{\text{Stuff}}$$

We set Stuff $\geqslant 0$

Then we solve this inequality.

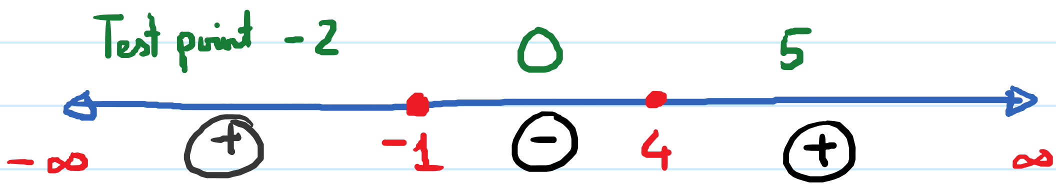
The solution set to this inequality will be the domain.

E.g. Find the domain

$$f(x) = \sqrt{x^2 - 3x - 4}$$

Step 1: Set $x^2 - 3x - 4 \geqslant 0$ means $\oplus$

Step 2: $(x-4)(x+1) \geqslant 0$ (Factor)



Solution to the inequality: $(-\infty, -1] \cup [4, \infty)$

$$\boxed{\text{Domain} = (-\infty, -1] \cup [4, \infty)}$$

E.g. Find domain $g(x) = \dfrac{1}{\sqrt{3x+1}}$

Sol: Set $3x + 1 > 0$

Note: If the expression that contain $\sqrt{}$ is in the denominator, the we require the stuff under the square root to be > 0. Reason: we cannot have "=" to 0 in the denominator.

Back to problem: Solve $3x + 1 > 0$

$$\rightarrow 3x > -1 \rightarrow x > -\frac{1}{3}$$

Solution to the inequality: $\left(-\frac{1}{3}, \infty\right)$

Domain $= \boxed{\left(-\frac{1}{3}, \infty\right)}$

3 Functions with mixed conditions:

E.g. Find the domain $f(x) = \dfrac{\sqrt{x+1}}{x^2 - 4x - 12}$.

Step 1: Set $x^2 - 4x - 12 = 0$

$$\rightarrow (x-6)(x+2) = 0$$

$\rightarrow x = 6$; $x = -2$. → these need to be excluded from domain

→ This takes care of the first requirement; i.e., denominator cannot be 0.

Step 2: Set $x+1 \geq 0$ (Reason: $x+1$ is under the square root)

$\longrightarrow x \geq -1.$

This takes care of the 2^{nd} requirement; i.e., requirement on Stuff under the square root.

Step 3: Combine the 2 requirements

So, Domain in interval notation

$$\boxed{[-1, 6) \cup (6, \infty)}$$

E.g. Find the domain of $f(x) = \dfrac{\sqrt{-9x+27}}{x-1}$

Step 1: Set $x - 1 = 0 \longrightarrow x = 1$ excluded from the domain

Step 2: Set $-9x + 27 \geq 0 \longrightarrow -9x \geq -27$

$\longrightarrow x \boxed{\leq} 3$ (Divide both sides by negative)

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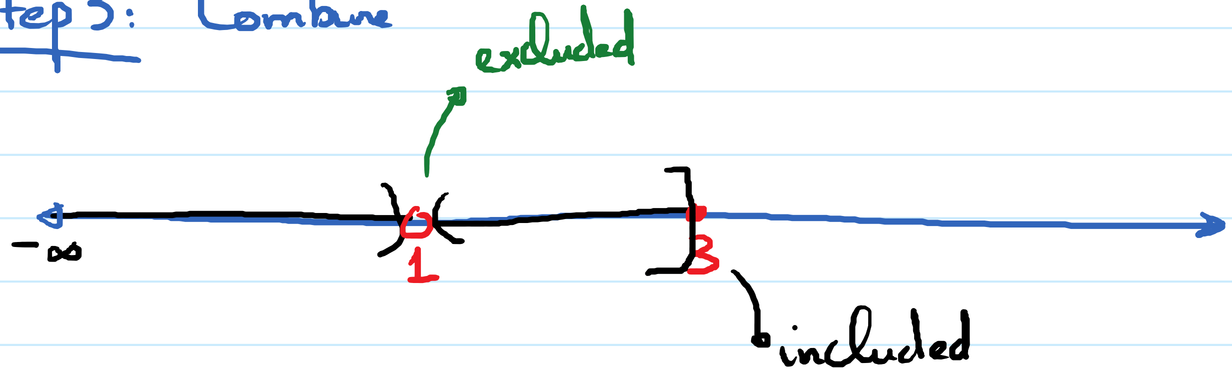
Step 3: Combine



Domain: $\boxed{(-\infty, 1) \cup (1, 3]}$

E.g. Find the domain of $g(x) = \dfrac{\sqrt{x^2 - 5x + 6}}{x^2 + 4}$.

Step 1: Set $x^2 + 4 = 0 \to x^2 = -4$

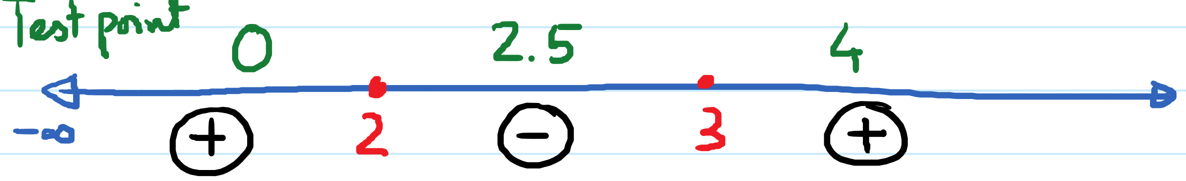
$\to x = \pm\sqrt{-4} = \pm 2i \to$ non-real

$\to$ nothing is excluded.

Step 2: Set $x^2 - 5x + 6 \geq 0$

$(x-2)(x-3) \geq 0$ means $\oplus$ and including

Test point



Solution to inequation: $(-\infty, 2] \cup [3, \infty)$

Domain $= \boxed{(-\infty, 2] \cup [3, \infty)}$

More practice.

Find the domain of the given function:

(a) $f(x) = \sqrt{6x^2 - x - 12}$ Ans: $\left(-\infty, -\frac{4}{3}\right] \cup \left[\frac{3}{2}, \infty\right)$

(b) $g(x) = \dfrac{5x + 7}{\sqrt{x^2 - 9}}$ Ans: $(-\infty, -3) \cup (3, \infty)$

(c) $h(x) = \dfrac{\sqrt{x - 5}}{x^2 - 121}$ Ans: $[5, 11) \cup (11, \infty)$