Exponential Functions

Wednesday, April 24, 2019 1:01 PM

Definition: An exponential function is a function where a number is raised to a variable.

$$f(x) = a^x \quad \text{where} \quad a > 0;\ a \neq 1$$

a: base

x: exponent (power)

E.g. Base $a = 2$. $f(x) = 2^x$

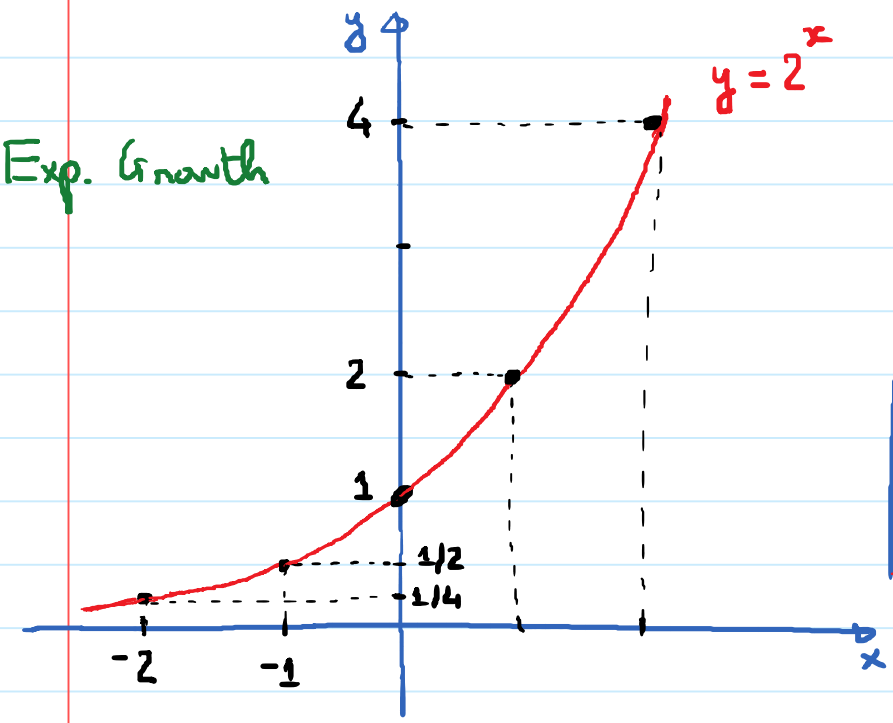
x	$f(x) = 2^x$	
-2	$1/4$	$\rightarrow (-2, \frac{1}{4})$
-1	$1/2$	$\rightarrow (-1, \frac{1}{2})$
0	1	$\rightarrow (0, 1)$
1	2	$\rightarrow (1, 2)$
2	4	$\rightarrow (2, 4)$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

$$2^0 = 1$$

Exp. Growth



Important Characteristics

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

y-intercept: $(0, 1)$

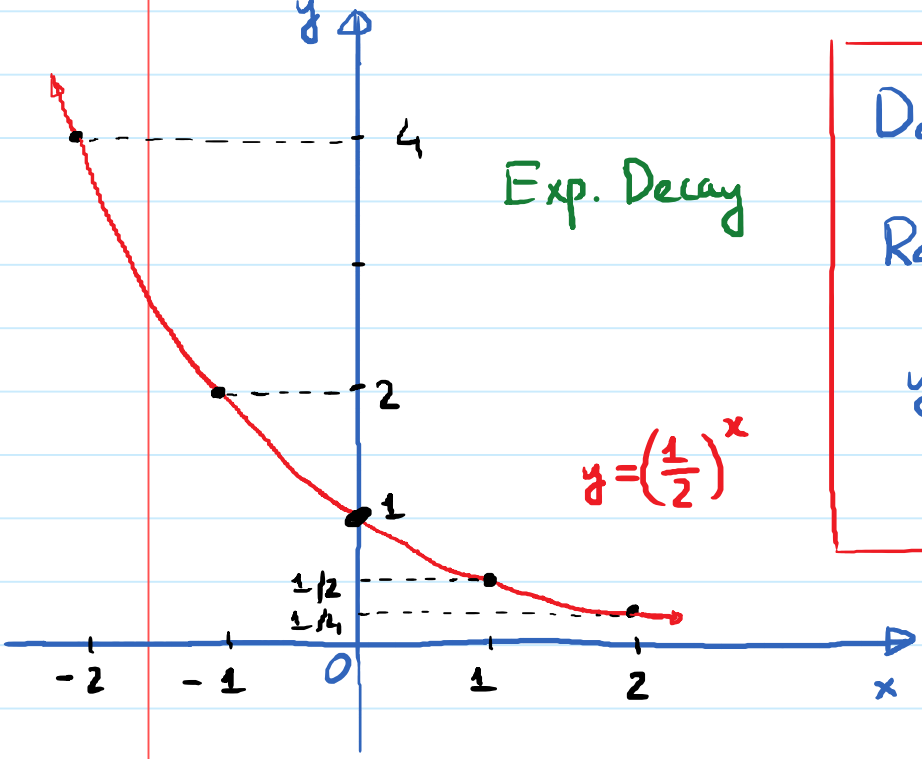
H.A.: $y = 0$

E.g. Base $a = \frac{1}{2}$ $\rightarrow$ $f(x) = \left(\frac{1}{2}\right)^x$

x	$f(x) = \left(\frac{1}{2}\right)^x$	
-2	4	$\rightarrow (-2, 4)$
-1	2	$\rightarrow (-1, 2)$
0	1	$\rightarrow (0, 1)$
1	$1/2$	$\rightarrow (1, 1/2)$
2	$1/4$	$\rightarrow (2, 1/4)$

$$\left(\frac{1}{2}\right)^{-2} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$$

$$\left(\frac{1}{2}\right)^{-1} = \frac{1}{\left(\frac{1}{2}\right)^1} = \frac{1}{\frac{1}{2}} = 2$$



Exp. Decay

$y = \left(\frac{1}{2}\right)^x$

Domain: $(-\infty, \infty)$

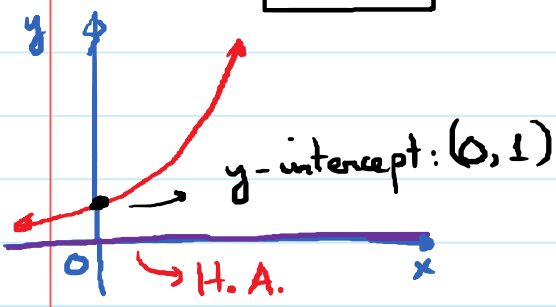
Range : $(0, \infty)$

y-intercept : $(0, 1)$

H.A. : $y = 0$

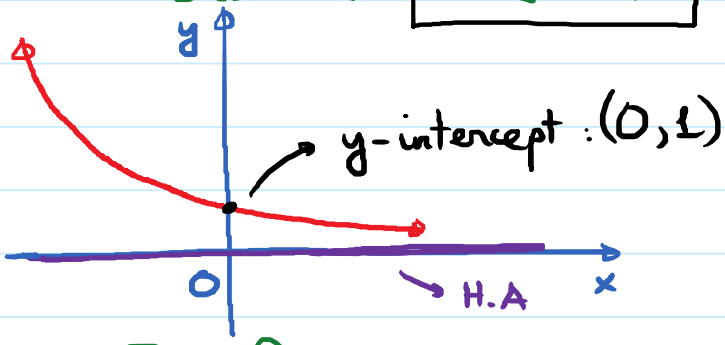
Summary of Behavior of $f(x) = a^x$

Base $a > 1$



Exp. Growth

Base a : $0 < a < 1$



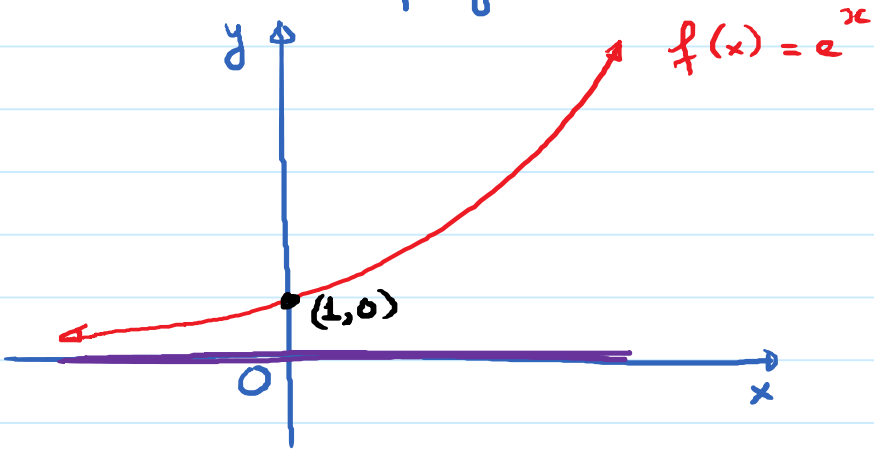
Exp. Decay

Special Base: Base $a = e$

where e is a mathematical constant: $e \simeq 2.71828...$

$$f(x) = e^x$$

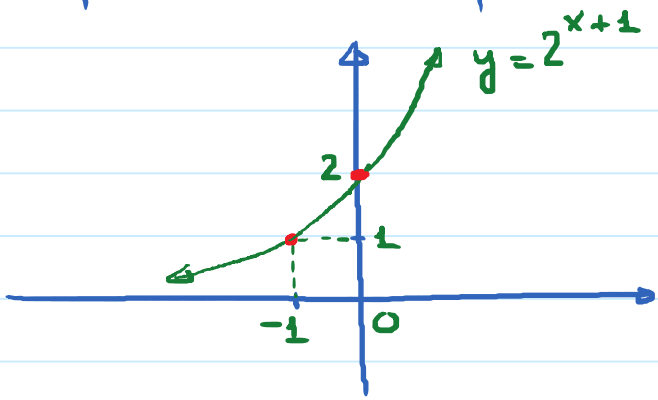
$e > 1 \rightarrow$ exp. growth.



Transformations of graphs of exp. functions.

E.g. $y = 2^{x+1}$. Parent function: $y = 2^x$

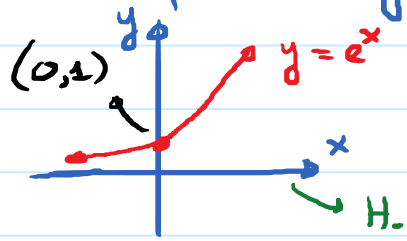
Transformation: Parent function moves left 1 unit.



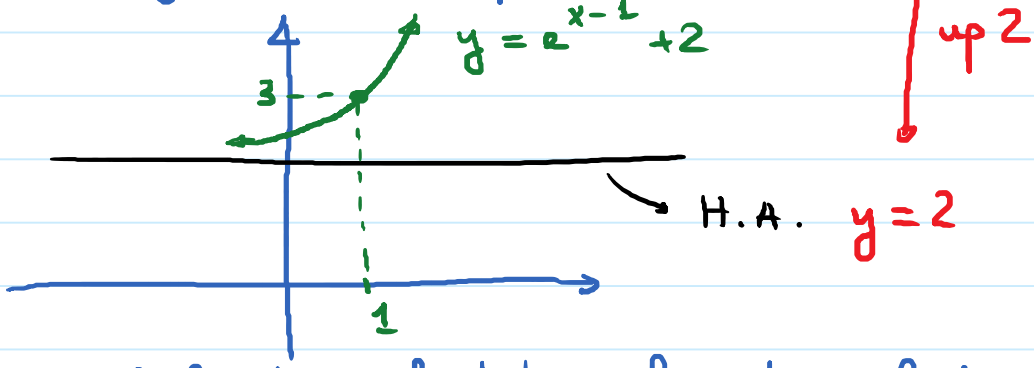
E.g. $y = 2^{x+2} - 3$. Parent function: $y = 2^x$

Transformation: Left 2 units, Down 3 units

E.g. $y = e^{x-1} + 2$ Parent function: $y = e^x$



Transformation: Right 1 unit, up 2 unit



E.g. Find parent function, find transformations, find where (0,1) goes, find H.A.

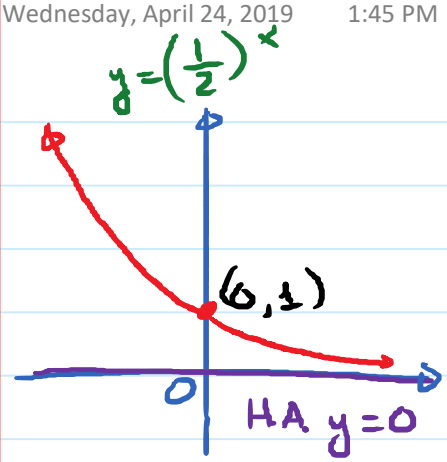
(a) $y = \left(\frac{1}{2}\right)^{x-1} - 1$.

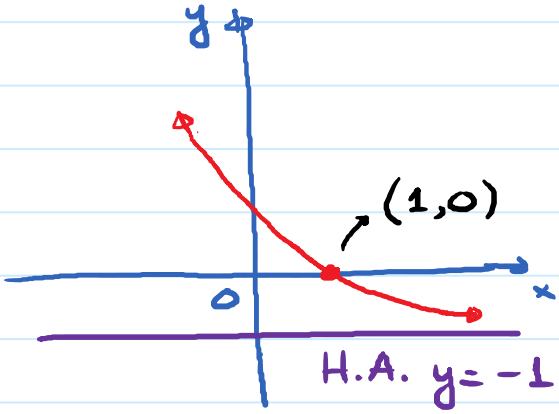
(b) $y = -e^{x+2} + 3$.

Sol:

(a) Parent function: $y = \left(\frac{1}{2}\right)^x$

Transformations: Right 1, Down 1





(b) Parent function: $y = e^x$

Transf: left 2, flip across x-axis, up 3.