Graphs of Higher Degree Polynomial Functions

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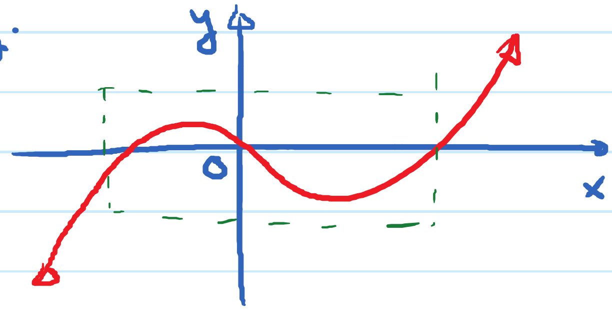
(1) End Behavior.

The end behavior is what the graph looks like at the very left and at the very right.

E.g.

Rises to the right



Falls to the left

How to determine the end behavior of polynomial functions from the formula?

Need.
- Degree: largest exponent of x in the function
- Leading coefficient: the # in front of term with largest exponent

degree

E.g. $f(x) = \boxed{-5}x^{\boxed{7}} + x^4 - 13x^2 + \frac{3}{5}x - 21.$

Degree: 7 leading coeff: -5

$g(x) = \boxed{4}\underbrace{(x-2)^3}_{x^3} \cdot \underbrace{(x+1)}_{x^1} \cdot \underbrace{(x-7)^2}_{x^2} = x^{6}$

leading coeff. degree

$$h(x) = \boxed{-\frac{1}{2}}\,\underbrace{(2x-3)^2}_{(2x)^2}\,(4x-1)(x+7)^4$$

$$\boxed{4}x^2 \cdot \boxed{4}x \quad \boxed{1}x^4$$

Degree = $\boxed{7}$

Leading coeff = $\boxed{-8}$

Leading Coeff. \ Degree	Even	Odd
Positive	↖ ↗	↙ ↗
Negative	↙ ↘	↖ ↘

E.g. $f(x) = 4x^6 - 5x^5 + 3x^4 - \frac{1}{2}x^3 + x^2 + x + 7$

Degree : 6 → $\boxed{\text{Even}}$. leading coeff: 4 → $\boxed{\text{Positive}}$

→ End behavior: rise to the right, rise to left

E.g. $g(x) = -\frac{1}{3}x^3 + 4x^2 - x + 17$

Degree: 3 → $\boxed{\text{Odd}}$ leading coeff: $-\frac{1}{3}$ → $\boxed{\text{negative}}$.

→ End Behavior: rise left, falls right

Why is this true?

$f(x) = \boxed{5x^4} + \dots$

→ Dominating term

$f(x) = \boxed{-5\cdot x^4} + \dots$

$f(x) = \boxed{5x^3} + \dots$

$f(x) = \boxed{-5x^3} + \dots$

Ex. Determine the end behavior of the function:

① $f(x) = (x-5)(x+1)^2(x-3)$ Degree: 4 → even; leading coeff: 1 → >0

② $g(x) = -3(x-5)^2(x+1)^3(x-3)^4$ Degree: 9 → odd; leading coeff: -3 → <0

2) Find x-intercept(s) (zeros) and their multiplicity:

How to find x-intercepts: Set $f(x) = 0$ then solve for x.

E.g. $f(x) = (x-1)^2 (x+5)(x-3)^3$

Find x-intercepts:

Set $f(x) = 0$: $0 = (x-1)^2 (x+5)(x-3)^3$

$\rightarrow (x-1)^2 = 0$ or $x+5 = 0$ or $(x-3)^3 = 0$

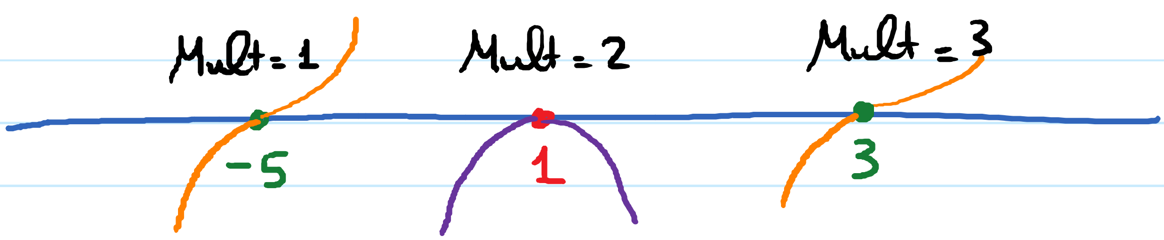
$\boxed{x = 1}$ or $\boxed{x = -5}$ or $\boxed{x = 3}$

Multiplicity = 2 Multiplicity = 1 Multiplicity = 3

x-intercepts: $(1,0)$; $(-5,0)$; $(3,0)$

Why is multiplicity important

odd mult. → crosses x-axis



even mult. → touches x-axis and bounces back

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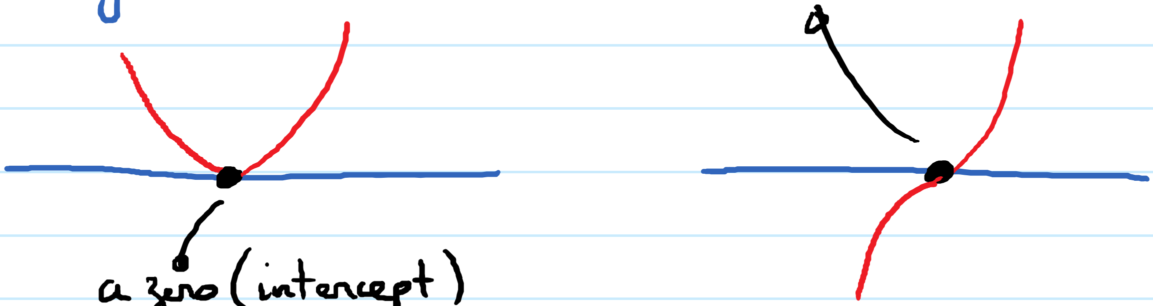
Summary:



③ Find y-intercept.

To find y-intercept: we plug $x=0$ into the function and simplify.

⟶ Use ①, ② and ③ to analyze graphs of polynomial functions.

E.g. $f(x) = x^3 - 9x$.

① End Behavior.

Degree: 3 (odd). leading coeff: 1 (positive).

⟶ End Behavior: Falls left, rises right.

② x-intercept(s) and multiplicity

$$x^3 - 9x = 0$$

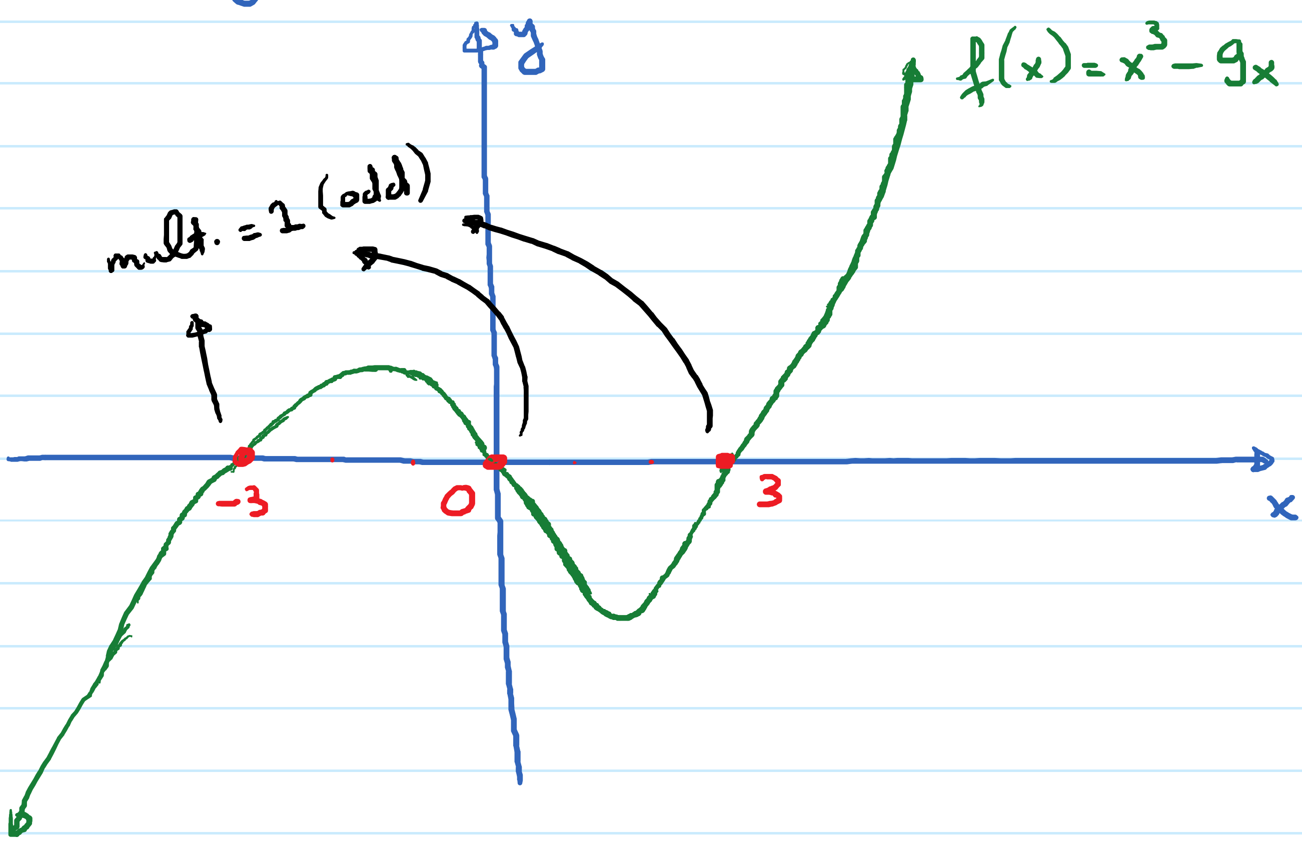
$$x(x^2 - 9) = 0$$

$$x(x+3)(x-3) = 0$$

Zeros:	0	-3	3
Mult.	1	1	1

③ y-intercept: $f(0) = (0)^3 - 9(0) = 0$

y-intercept: $(0,0)$



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Ex. $g(x) = -\frac{1}{2}x(x+3)(x-1)^2$

Determine end behavior, zeros and multiplicity, y-intercept and graph the function.

① End Behavior:

Degree : 4 — Even

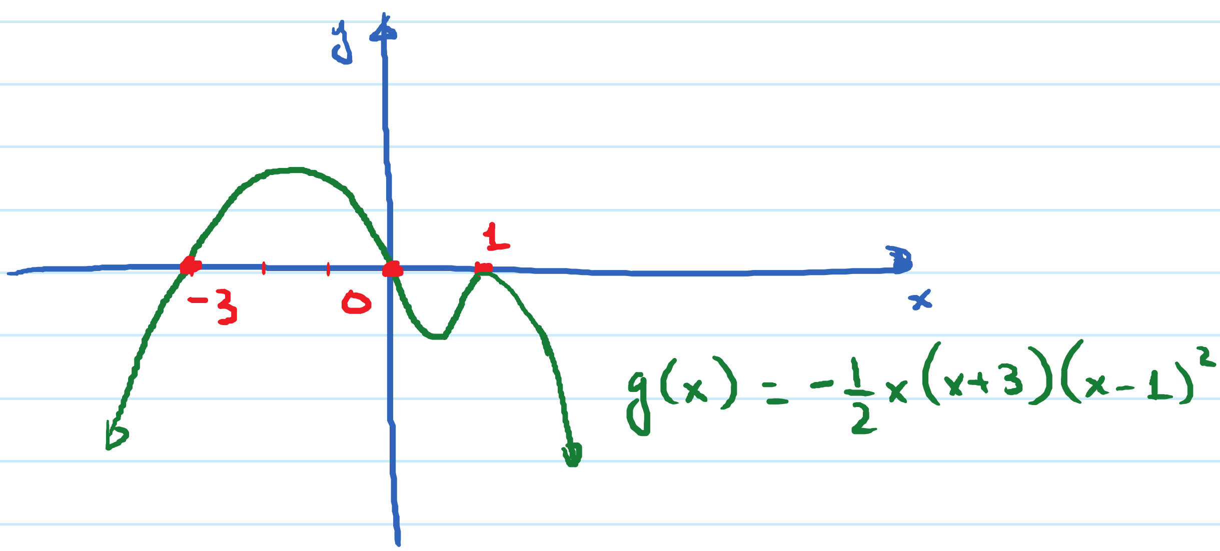
leading coeff: $-\frac{1}{2}$ — Negative

→ End Behavior.

② Zeros and Multiplicity.

	0	−3	1
Mult.	1	1	2

③ y-intercept (0, 0)



Ex. Same question

$h(x) = x^4 + 14x^3 + 49x^2$

① End Behavior: Degree = 4 (even), leading coeff = 1 (positive) $\rightarrow$

② $x^2(x^2 + 14x + 49) = 0$

$x^2(x+7)^2 = 0$

Zeros: 0, -7

Mult. 2 2

③ y-intercept: (0,0)