# Inverse Functions

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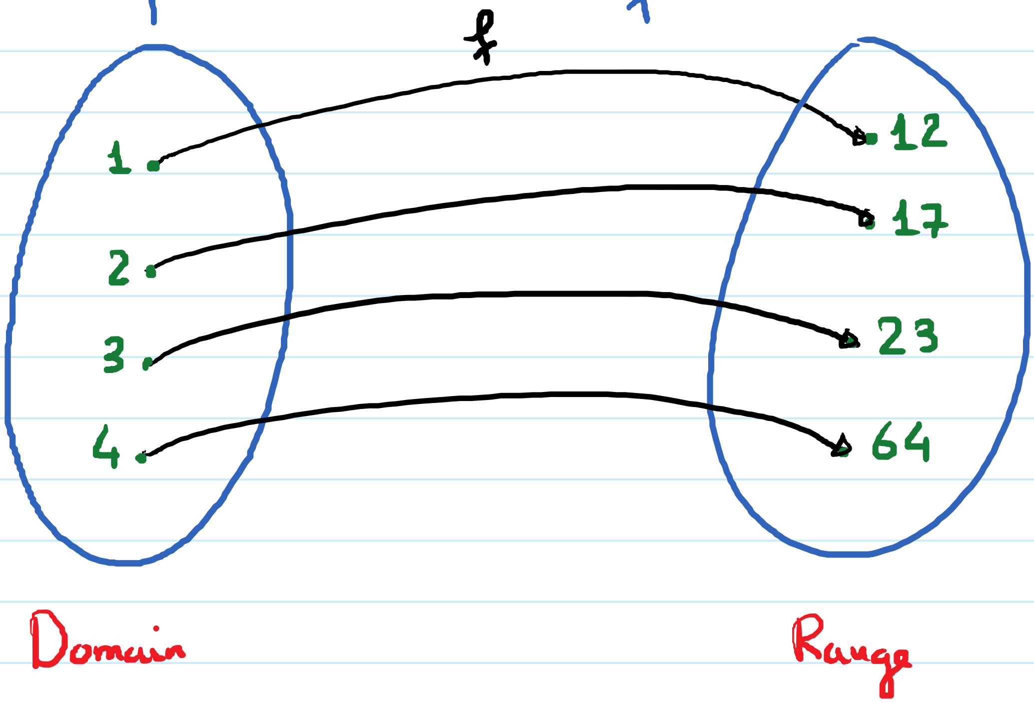
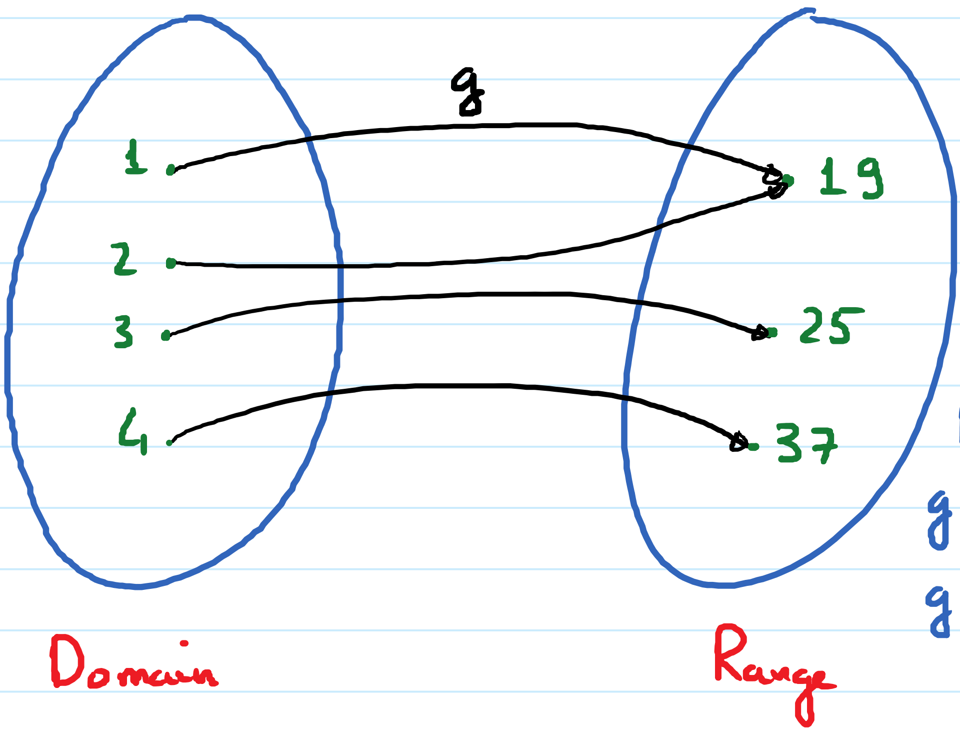
## One-to-one functions:

A function is one-to-one if every value of y corresponds to one value of x.




f is a one-to-one function.




g is NOT a one-to-one function b/c.

$g(1) = 19$

$g(2) = 19$

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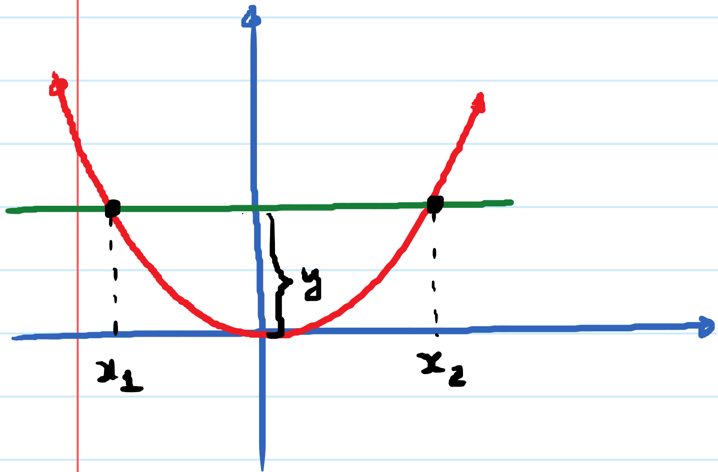
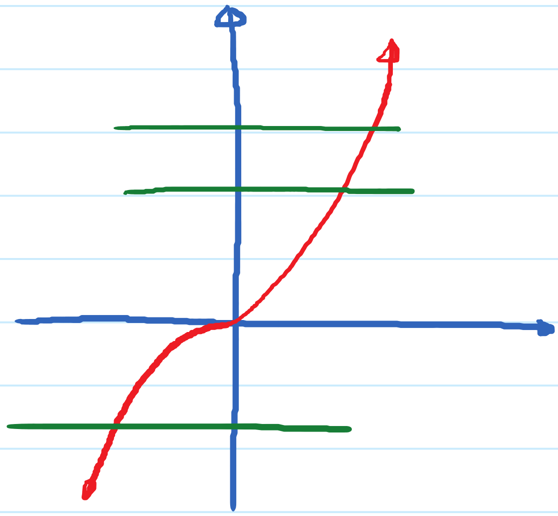
We can use the Horizontal Line Test to determine if a graph is the graph of a one-to-one function.




NOT one-to-one.

( there are more than one $x$ that corresponds to a $y$



One-to-one.

(Every horizontal line intersects exactly once)

One-to-one functions have inverses.

**Q:** How do we determine if 2 functions are inverses of one another.

**A:** If $f$ and $g$ are functions, then $f$ and $g$ are inverses of one another if:

$$\boxed{f(g(x)) = x \quad ; \quad g(f(x)) = x}$$

E.g. $f(x) = \frac{1}{4}\boxed{x} - 5$ and $g(x) = 4x + 20$.

$$f(g(x)) = \frac{1}{4}(4x+20) - 5$$
$$= x + 5 - 5 = x$$
$$g(f(x)) = 4\left(\frac{1}{4}x - 5\right) + 20$$
$$= x - 20 + 20 = x$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, So $f$ and $g$ are inverses of one-another.

E.g. $f(x) = \sqrt[3]{x+5}$ ; $g(x) = x^3 - 5$

$$f(g(x)) = \sqrt[3]{(x^3-5)+5} = \sqrt[3]{x^3} = x$$
$$g(f(x)) = \left(\sqrt[3]{x+5}\right)^3 - 5 = x + 5 - 5 = x.$$

So, $f$ and $g$ are inverses of one another.

Q: How do we find the inverse function of a one-to-one function?

Illustrate this process by an example.

E.g. Given $f(x) = \frac{1}{7}x + 5$ (one-to-one)

Q: Find the inverse function $\boxed{f^{-1}(x)}$ of $f$

Notation for the inverse function of $f$.

Step 1: Replace the notation $f(x)$ by $y$ in the formula for $f$:

$$y = \frac{1}{7}x + 5$$

Step 2: Solve for $x$ in terms of $y$ (get $x$ by itself) from the above equation.

$\longrightarrow$ $y - 5 = \frac{1}{7}x$ (Subtract 5 from both sides)

$\longrightarrow$ $7(y-5) = x$ (Multiply both sides by 7)

$$7y - 35 = x$$

$$\longrightarrow x = 7y - 35$$

Step 3: Swap x and y in the equation from step 2.

$$y = 7x - 35.$$

Step 4: Replace y by the notation $f^{-1}(x)$.

$$\boxed{f^{-1}(x) = 7x - 35} \longrightarrow \text{Answer}$$

This is the formula for the inverse function of f.

Ex: Find the inverse function of the given one-to-one function:

1. $f(x) = 2x^3 + 7$

2. $g(x) = \dfrac{5}{x+3}$

Sol: 1 Step 1: Replace $f(x)$ by $y$.

$$y = 2x^3 + 7.$$

Step 2: Get $x$ by itself:

$$y - 7 = 2x^3$$

$$\rightarrow \frac{y-7}{2} = x^3$$

$$\rightarrow x^3 = \frac{y-7}{2} \rightarrow x = \sqrt[3]{\frac{y-7}{2}}$$

Step 3: Swap $x$ and $y$.

$$y = \sqrt[3]{\frac{x-7}{2}}$$

Step 4: Replace $y$ by $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x-7}{2}}}$$

2 Step 1: Replace $g(x)$ by $y$.

$$y = \frac{5}{x+3}$$

Step 2: Get $x$ by itself.

$$y(x+3) = 5$$ (Multiply both sides by $x+3$)

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$$x + 3 = \frac{5}{y} \quad \text{(Divide both sides by } y\text{)}$$

$$x = \frac{5}{y} - 3$$

**Step 3:** Swap x and y

$$y = \frac{5}{x} - 3.$$

**Step 4:** Replace y by $g^{-1}(x)$

$$\boxed{g^{-1}(x) = \frac{5}{x} - 3}$$

**Ex:** Find the inverse of the given function.

① $f(x) = \frac{1}{3}\sqrt{x-4}$

② $g(x) = \frac{5-7x}{x} + 1.$

**Sol:**

① $y = \frac{1}{3}\sqrt{x-4}$ (Step 1)

$\rightarrow 3y = \sqrt{x-4} \rightarrow 9y^2 = x - 4$

$\rightarrow 9y^2 + 4 = x \rightarrow x = 9y^2 + 4$

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$$y = 9x^2 + 4 \longrightarrow \boxed{f^{-1}(x) = 9x^2 + 4}$$

2 $$y = \frac{5-7x}{x} + 1$$

$$y - 1 = \frac{5-7x}{x}$$

$$\longrightarrow x(y-1) = 5 - 7x$$

$$\longrightarrow xy - x = 5 - 7x$$

$$\longrightarrow xy - x + 7x = 5$$

$$\longrightarrow xy + 6x = 5$$

$$\longrightarrow x(y+6) = 5$$

$$\longrightarrow x = \frac{5}{y+6}$$

$$\longrightarrow y = \frac{5}{x+6} \longrightarrow \boxed{g^{-1}(x) = \frac{5}{x+6}}$$