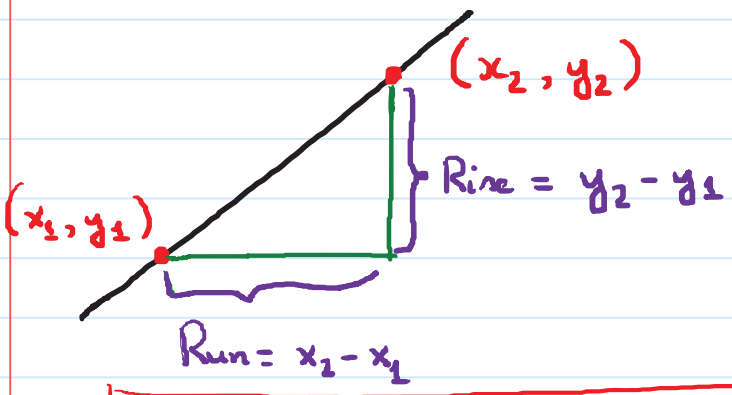
Linear Functions

Monday, February 4, 2019 1:02 PM

Slope



$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of the line that passes through the points (x_1, y_1) and (x_2, y_2)

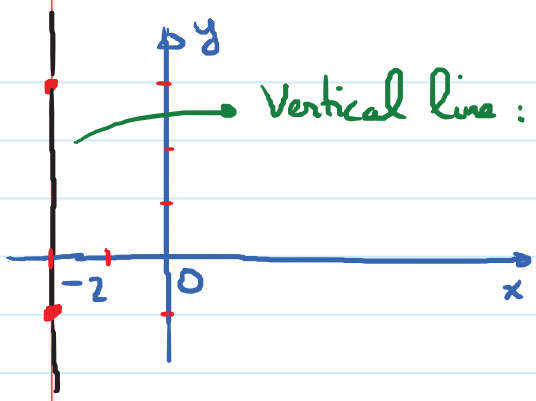
E.g. Line passes through $(\underbrace{-3}_{x_1}, \underbrace{4}_{y_1})$ and $(\underbrace{9}_{x_2}, \underbrace{1}_{y_2})$

Slope = ?

$$\text{Slope} = \frac{1-4}{9-(-3)} = \frac{-3}{12} = \boxed{-\frac{1}{4}}$$

E.g. Points: $(-2, 3)$ and $(-2, 1)$

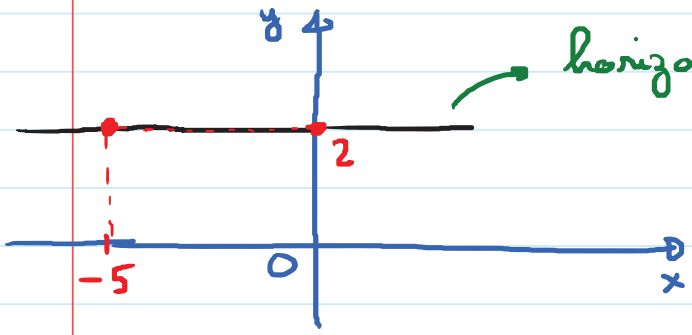
$$\text{Slope} = \frac{1-3}{-2-(-2)} = \frac{-2}{-2+2} = \frac{-2}{0} = \text{undefined.}$$



Vertical line: Slope is undefined

E.g. Points: $(-5,2)$ and $(0,2)$

$$\text{Slope} = \frac{2-2}{0-(-5)} = \frac{0}{5} = 0$$

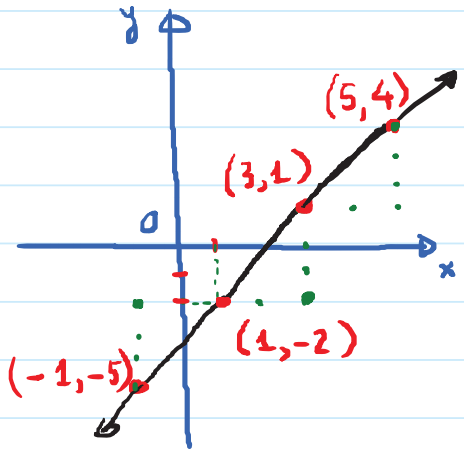


Horizontal line: Slope = 0

Graph a line knowing the slope and a point.

E.g. Graph the line with slope $\boxed{m = \frac{3}{2}}$ and it passes through $(1,-2)$.

$\frac{\text{Rise}}{\text{Run}}$



Step 1: Start with given point.

Step 2: Use the slope to find the next point(s) $\left(\text{Slope} = \frac{\text{Rise}}{\text{Run}}\right)$

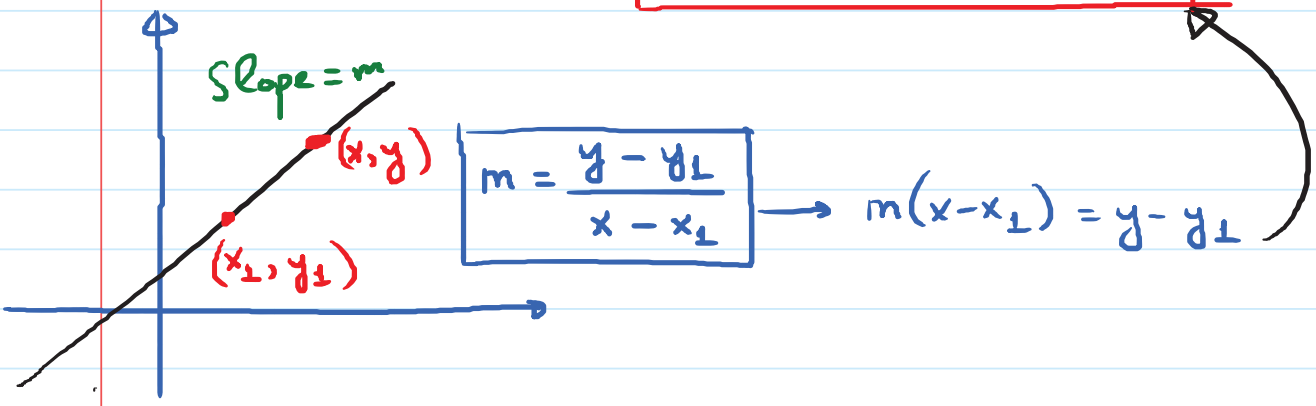
Step 3: Connect the points

Find the equations of a line:

Slope-intercept form: $\boxed{y = mx + b}$

m: slope; b: y-intercept: $(0, b)$

Point-Slope form: $\boxed{y - y_1 = m(x - x_1)}$



$\boxed{m = \frac{y - y_1}{x - x_1}} \rightarrow m(x - x_1) = y - y_1$

E.g. Find the slope-intercept form of the line with slope $m = \frac{3}{5}$ and it passes through $(1, -2)$

1st way of solving this:

find b?

$y = mx + b \rightarrow y = \frac{3}{5}x + b$

$m = \frac{3}{5}$; $y = -2$; $x = 1$

$\rightarrow -2 = \frac{3}{5} \cdot 1 + b \rightarrow b = -2 - \frac{3}{5}$

$\boxed{b = -\frac{13}{5}}$

Slope-intercept equation:

$$\boxed{y = \frac{3}{5}x - \frac{13}{5}}$$

2nd way of doing this:

Start with point-slope equation:

$$y - y_1 = m(x - x_1)$$

$y_1 = -2$, $m = \frac{3}{5}$, $x_1 = 1$

$$y - (-2) = \frac{3}{5}(x - 1)$$

$$y + 2 = \frac{3}{5}x - \frac{3}{5}$$

$$y = \frac{3}{5}x - \frac{3}{5} - 2$$

Simplify

$$\boxed{y = \frac{3}{5}x - \frac{13}{5}}$$

E.g. Find the slope-intercept equation of the line that passes through $(-3, 4)$ and $(9, 1)$

Step 1: Find the slope

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{9 - (-3)} = \frac{-3}{12} = \boxed{-\frac{1}{4}}$$

Step 2: Use either form to get equation.

$y = -\frac{1}{4}x + b$ (Pick $(9,1)$)

y: 1; x: 9

$1 = -\frac{1}{4} \cdot 9 + b \rightarrow b = 1 + \frac{9}{4} = \frac{13}{4}$.

Slope intercept equation: $\boxed{y = -\frac{1}{4}x + \frac{13}{4}}$

E.g. Find the formula for the linear function

$y = f(x)$ given: $f(6) = 3$; $f(-2) = 4$

$f(6) = 3$: point $(6,3)$ $f(-2) = 4$: point $(-2,4)$

Sol:

$$\text{Slope} = \frac{4-3}{-2-6} = \frac{1}{-8} = -\frac{1}{8}.$$

Point-Slope Form:

$y - y_1 = m(x - x_1)$

y_1: 3; m: $-\frac{1}{8}$; x_1: 6

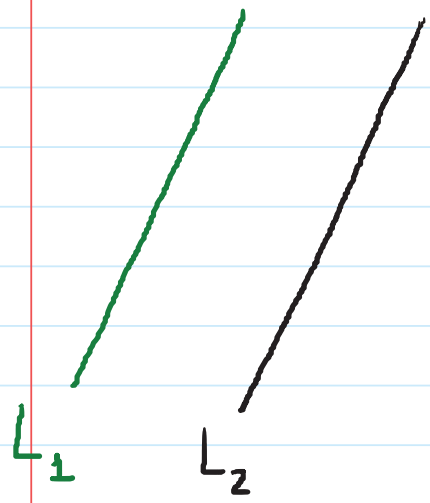
$y - 3 = -\frac{1}{8}(x-6)$

$$y - 3 = -\frac{1}{8}x + \frac{3}{4}$$

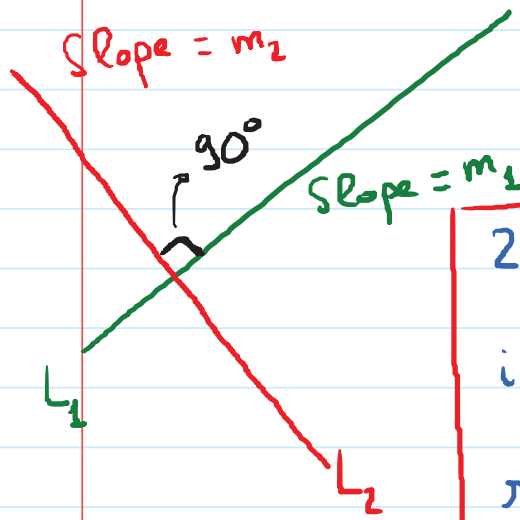
$$y = -\frac{1}{8}x + \frac{3}{4} + 3$$

$$\boxed{y = -\frac{1}{8}x + \frac{15}{4}}$$

Parallel and Perpendicular Lines



2 lines are parallel if and only if they have the same slope.



Assume: lines are not vertical / horizontal

2 lines are perpendicular if and only if the slope of one is the negative reciprocal of the slope of the other.

$$\boxed{m_2 = -\frac{1}{m_1}}$$

E.g. Given the line: $L_1: 5x - 2y = 13$.

(a) Find the equation of the line L_2 that passes through $(-3, -5)$ and is parallel to L_1.

(b) Find the equation of the line L_2 that passes through $(-3, -5)$ and is perpendicular to L_1.

Sol

(a) $L_1: 5x - 2y = 13$

$$-2y = -5x + 13$$

$$y = \frac{5}{2}x - \frac{13}{2}$$

→ Slope of L_1: $\frac{5}{2}$.

Since L_2 is parallel to L_1, slope of L_2 is $\frac{5}{2}$

Slope-intercept of L_2: $y = \frac{5}{2}x + b$.

(y → -5, x → -3)

$$-5 = \frac{5}{2}(-3) + b$$

$$\rightarrow -5 = -\frac{15}{2} + b.$$

$$b = -5 + \frac{15}{2} = \frac{5}{2}$$

So, the Slope-intercept equation of L_2 is

L_2: $\boxed{y = \frac{5}{2}x + \frac{5}{2}}$

(b) Slope of L_1 is : $\frac{5}{2}$

Since L_2 is perpendicular to L_1, slope of L_2 is:

$-\frac{2}{5}$.

L_2: $y = -\frac{2}{5}x + b$

$y = -5$, $x = -3$

$-5 = -\frac{2}{5}(-3) + b$

$-5 = \frac{6}{5} + b \rightarrow b = -5 - \frac{6}{5} = -\frac{31}{5}$.

So, L_2 : $\boxed{y = -\frac{2}{5}x - \frac{31}{5}}$