Logarithmic Function.

Wednesday, April 24, 2019 1:54 PM

Definition of the logarithmic function.

Consider the number $a > 0$, $a \neq 1$.

number

$\boxed{y} = \underbrace{\log_{\boxed{a}} \boxed{x}}_{\text{read as log base a of x}}$ gives us the exponent y such that a^y is equal to x

base

exp.

In other words,

$$y = \log_a x \text{ is equivalent to } a^y = x.$$

E.g. Base $a = 2$.

\#

$\log_{\boxed{2}} \boxed{2} = \boxed{1} \rightarrow$ exp ; \# $\log_{\boxed{2}} \boxed{4} = \boxed{2}$ exp

base base

\#

$\log_{\boxed{2}} \boxed{8} = \boxed{3}$ exp ; $\log_2 16 = 4$

base

$\log_2 32 = 5$; $\log_2 64 = 6$

E.g. $\log_3 9 = 2$; $\log_3 1 = 0$

$\log_3 \left(\frac{1}{3}\right) = -1$; $\log_3 \left(\frac{1}{9}\right) = -2$.

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Notation: If the base a is equal to e, that is, we have $\log_e x$, then we write: $\ln x$.

So, $y = \ln x$ means the base is the # e.

If the base a is equal to 10, that is, we have $\log_{10} x$, then we write $\log x$

So, $y = \log x$ means the base is the # 10.

Basic Properties of Log function:

$$\log_{\underset{\text{base}}{\boxed{a}}} a = \overset{\text{exp}}{\boxed{1}} \quad (\text{Reason: } a^1 = a)$$

$$\log_a 1 = 0 \quad (\text{Reason: } a^0 = 1)$$

$$\log_a a^x = x \quad (\text{Reason: log and exp functions are inverses})$$

$$a^{\log_a x} = x \quad (\text{Same reason as above})$$

If the base is e, all the above equations become:

$$\boxed{\ln e = 1 \;;\quad \ln 1 = 0 \;;\quad \ln e^x = x \;;\quad e^{\ln x} = x}$$

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Convert from Logarithmic Form to Exponential form and vice versa.

* Exp Form ⟶ Log Form

E.g. $\underset{\text{base}}{\boxed{4}}^{\overset{\text{exp}}{\boxed{3}}} = \underset{\#}{\boxed{64}} \longrightarrow \log_{\underset{\text{base}}{\boxed{4}}} \overset{\#}{\boxed{64}} = \overset{\text{exp}}{\boxed{3}}$

$$\underset{\text{base}}{\boxed{8}}^{\overset{\text{exp}}{\boxed{0}}} = \underset{\#}{\boxed{1}} \longrightarrow \boxed{\log_8 1 = 0}$$

$$\sqrt[3]{8} = 2 \underset{\text{Rewrite cube root}}{\longrightarrow} (\underset{\text{base}}{\boxed{8}})^{\overset{\text{exp}}{\boxed{\frac{1}{3}}}} = \overset{\#}{\boxed{2}} \longrightarrow \boxed{\log_8 2 = \frac{1}{3}}$$

$$\underset{\text{base}}{\boxed{7}}^{\overset{\text{exp}}{\boxed{y}}} = \underset{\#}{\boxed{300}} \longrightarrow \boxed{\log_7 300 = y}$$

$$\underset{\text{base}}{\boxed{b}}^{\overset{\text{exp}}{\boxed{\frac{1}{2}}}} = \underset{\#}{\boxed{\sqrt{17}}} \longrightarrow \boxed{\log_b \sqrt{17} = \frac{1}{2}}$$

$$\underset{\text{base}}{\boxed{e}}^{\overset{\text{exp}}{\boxed{x}}} = \underset{\#}{\boxed{12}} \longrightarrow \log_e 12 = x \underset{\text{Rewrite using correct notation}}{\rightarrow} \boxed{\ln 12 = x}$$

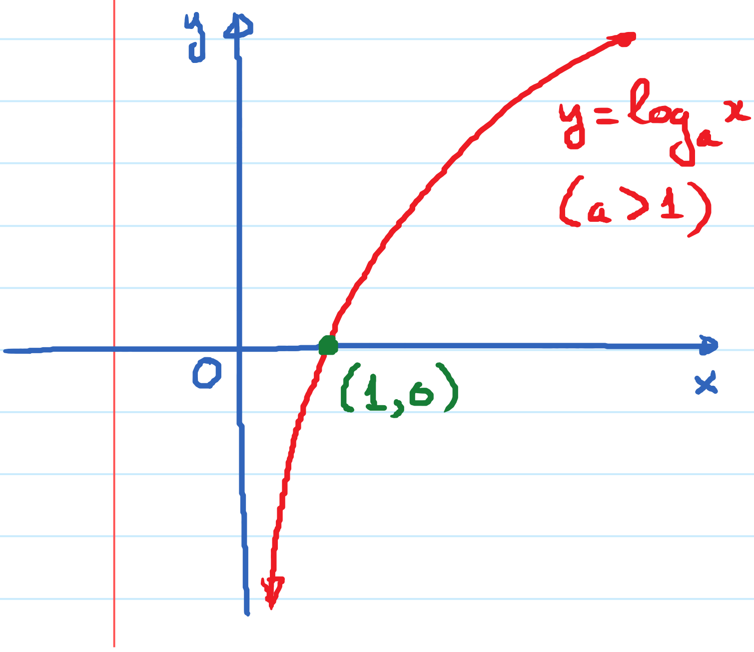
* Log Form ⟶ Exp Form.

$$\log_{\underset{\text{base}}{\boxed{9}}} \overset{\#}{\boxed{3}} = \overset{\text{exp}}{\boxed{\frac{1}{2}}} \longrightarrow \boxed{(9)^{\frac{1}{2}} = 3}$$

$$\underset{\text{base} = e}{\ln} \overset{\#}{\boxed{e}} = \overset{\text{exp}}{\boxed{1}} \longrightarrow \boxed{e^{1} = e}$$

$$\underset{\text{base} = 10}{\log} \overset{\#}{\boxed{1000}} = \overset{\text{exp}}{\boxed{3}} \longrightarrow \boxed{10^{3} = 1000}$$

$$\log_{\underset{\text{base}}{\boxed{b}}} \overset{\#}{\boxed{125}} = \overset{\text{exp}}{\boxed{y}} \longrightarrow \boxed{b^{y} = 125}$$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

Vertical Asymptote: $x = 0$

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Down 2 unit.

E.g. $y = \log_3(\underbrace{x+1}_{\text{Left by 1 unit}}) \boxed{-2}$

→ V.A.: $x = -1$.

Key point: $(1,0) \rightarrow$ left 1, down 2 : $(0,-2)$