

Miscellaneous Equations

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12:59 PM

I Solving Polynomial Equations by Factoring.

E.g. $6x^3 + 2x^2 - 8x = 0$ (Degree 3 equation)

$$2x(3x^2 + x - 4) = 0 \quad (\text{Factor})$$

$$2x = 0$$

$$\downarrow$$
$$x = 0$$

on $3x^2 + x - 4 = 0$ (Zero Product Property)

$$\begin{array}{ccc} & -12 & \\ \frac{4}{3} & \times & -\frac{3}{3} \\ & 1 & \end{array} \rightarrow \begin{array}{ccc} & & \\ \frac{4}{3} & \times & -\frac{1}{1} \\ & & \end{array}$$

$$(3x + 4)(x - 1) = 0$$

$$3x + 4 = 0 \quad \text{on} \quad x - 1 = 0$$

$$\downarrow$$
$$x = -\frac{4}{3}$$

$$\downarrow$$
$$x = 1$$

Solution set: $\left\{0, -\frac{4}{3}, 1\right\}$

E.g. Solve: $2x^3 - 5x^2 - 2x = -5$

$$2x^3 - 5x^2 - 2x + 5 = 0 \quad (\text{Add 5 to both sides})$$

$$x^2(2x - 5) - 1(2x - 5) = 0 \quad (\text{Factor by grouping})$$

$$(2x - 5)(x^2 - 1) = 0$$

$$(x-1)(x+1)=0$$

$$2x - 5 = 0 \quad \text{or} \quad x^2 - 1 = 0$$

$$x = \frac{5}{2}$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\text{Solution set: } \left\{ \frac{5}{2}, 1, -1 \right\}$$

II) Solve Radical Equations.

E.g. $\sqrt{2x-5} - 6 = 0$. Solve.

$$\sqrt{2x-5} = 6 \quad (\text{Add 6 to both sides to isolate the radical})$$

$$(\sqrt{2x-5})^2 = (6)^2 \quad (\text{Square both sides})$$

$$2x - 5 = 36$$

$$2x = 41$$

$$x = \frac{41}{2} \rightarrow \text{this is, in fact, a solution}$$

Check: Plug $x = \frac{41}{2}$ into original equation:

$$\sqrt{2 \cdot \left(\frac{41}{2}\right) - 5} - 6 \stackrel{?}{=} 0 \quad \left| \begin{array}{l} \sqrt{41-5} - 6 \stackrel{?}{=} 0 \\ \sqrt{36} - 6 \stackrel{?}{=} 0 \\ 6 - 6 = 0 \checkmark \end{array} \right.$$

Ex. Solve: $\sqrt{x+1} + x = 5$

$\sqrt{x+1} = 5 - x$ (Isolate the radical)

$(\sqrt{x+1})^2 = (5-x)^2$ (Square both sides)

$x+1 = 25 - 10x + x^2$

$0 = x^2 - 11x + 24$

$x^2 - 11x + 24 = 0$

~~$\begin{array}{ccc} & 24 & \\ -8 & & -3 \\ & -11 & \end{array}$~~

$(x-8)(x-3) = 0$

$x = 8 ; x = 3$

Check: ~~$x=8$~~

Not a solution

$\sqrt{8+1} + 8 \stackrel{?}{=} 5$

$\sqrt{9} + 8 \stackrel{?}{=} 5$

$3 + 8 \stackrel{?}{=} 5$

$x=3$: $\sqrt{3+1} + 3 \stackrel{?}{=} 5$

A solution $\sqrt{4} + 3 \stackrel{?}{=} 5$

$2 + 3 = 5$ ✓

Solution set: $\boxed{\{3\}}$

III Equations with rational exponent.

$$\sqrt{A} = A^{\frac{1}{2}} ; \sqrt[3]{A} = A^{\frac{1}{3}}$$

$$\sqrt[3]{A^2} = A^{\frac{2}{3}}$$

In general, rational exponent:

$$A^{\frac{m}{n}} = \begin{matrix} \nearrow \sqrt[n]{A^m} \\ \searrow (\sqrt[n]{A})^m \end{matrix}$$

E.g. $(2x+3)^{\frac{3}{2}} = 8$

Raise both sides to the exponent

$$\boxed{\frac{2}{3}}$$

reciprocal of $\frac{3}{2}$

$$\left[(2x+3)^{\frac{3}{2}} \right]^{\frac{2}{3}} = (8)^{\frac{2}{3}}$$

$$(A^p)^q = A^{p \cdot q}$$

$$2x+3 = (\sqrt[3]{8})^2$$

$$2x+3 = (2)^2 \rightarrow 2x+3 = 4 \rightarrow 2x = 1$$

$$x = \frac{1}{2}$$

Check:

$$\left(2 \cdot \frac{1}{2} + 3\right)^{\frac{3}{2}} \stackrel{?}{=} 8$$

$$(1 + 3)^{\frac{3}{2}} \stackrel{?}{=} 8$$

$$(4)^{\frac{3}{2}} \stackrel{?}{=} 8$$

$$(\sqrt{4})^3 \stackrel{?}{=} 8 \rightarrow (2)^3 = 8 \quad \checkmark$$

Solution set is: $\left\{\frac{1}{2}\right\}$

E.g. $(x^2 + 6x - 7)^{\frac{3}{2}} = 27$

Raise both sides to the $\frac{2}{3}$

$$\left[(x^2 + 6x - 7)^{\frac{3}{2}}\right]^{\frac{2}{3}} = (27)^{\frac{2}{3}}$$

$$x^2 + 6x - 7 = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$$

$$x^2 + 6x - 7 = 9$$

$$x^2 + 6x - 16 = 0$$

~~$$\begin{array}{ccc} & -16 & \\ -2 & & 8 \\ & 6 & \end{array}$$~~

$$(x - 2)(x + 8) = 0$$

$$x = 2 \text{ or } x = -8$$