

Check: $x=2$: $\left[(2)^2 + 6 \cdot 2 - 7 \right]^{\frac{3}{2}} \stackrel{?}{=} 27$
a solution

$$(9)^{\frac{3}{2}} \stackrel{?}{=} 27$$

$$(\sqrt{9})^3 \stackrel{?}{=} 27$$

$$(3)^3 = 27 \quad \checkmark$$

$x=-8$: $\left[(-8)^2 + 6 \cdot (-8) - 7 \right]^{\frac{3}{2}} \stackrel{?}{=} 27$
a solution

$$(9)^{\frac{3}{2}} = 27 \quad \checkmark$$

Solution set: $\{2, -8\}$

Ex. Solve: $(2x - 5)^{\frac{2}{5}} = 9$

$$2x - 5 = (9)^{\frac{5}{2}} = (\sqrt{9})^5 = (3)^5 = 243$$

$$2x = 248$$

$$x = 124.$$

Check: HW.

III Equations that are quadratic in form.

Solve: $x^6 - 7x^3 - 8 = 0$

Let $u = x^3$. Then: $u^2 = (x^3)^2 = x^6$

After this u -substitution, the original equation

becomes: $u^2 - 7u - 8 = 0$

~~$\begin{array}{cc} & -8 \\ -8 & & 1 \\ & -7 \end{array}$~~

$(u - 8)(u + 1) = 0$

$u = 8 ; u = -1$

So, this tells us that

$x^3 = 8$



$x = \sqrt[3]{8}$



$x = 2$

or $x^3 = -1$



$x = \sqrt[3]{-1}$



$x = -1$

Ex. Solve: $x - 2\sqrt{x} - 8 = 0$

$$-2\sqrt{x} = -x + 8$$

$$\sqrt{x} = \frac{1}{2}x - 4 \quad (\text{Isolate radical})$$

$$x = \left(\frac{1}{2}x - 4\right)^2 \quad (\text{Square both sides})$$

$$x = \frac{1}{4}x^2 - 2 \cdot \frac{1}{2}x \cdot 4 + 16$$

$$x = \frac{1}{4}x^2 - 4x + 16$$

$$0 = \frac{1}{4}x^2 - 5x + 16$$

$$0 = x^2 - 20x + 64$$

$$\begin{array}{ccc} & 64 & \\ -16 & \times & -4 \\ & -20 & \end{array}$$

$$0 = (x - 16)(x - 4)$$

$$x = 4; x = 16 \rightarrow \text{check!}$$

2nd way to solve:

$$u^2 \rightarrow \boxed{x} - 2\boxed{\sqrt{x}} - 8 = 0$$

$\swarrow$ u

Let $u = \sqrt{x}$. Then $u^2 = x$

Equation becomes: $u^2 - 2u - 8 = 0$

$$\begin{array}{cc} & -8 \\ -4 & \times & 2 \\ & -2 \end{array}$$

$$(u-4)(u+2) = 0$$

$$u = 4 ; u = -2.$$

$$\sqrt{x} = 4 ; \sqrt{x} = -2.$$

$$x = 16 ; x = 4 \rightarrow$$

HW: Check solution