Operations on Functions

Monday, April 15, 2019 1:09 PM

1 Combination of Functions: $f(x)$, $g(x)$ are functions.

* Sum of f and g: $(f+g)(x) = f(x) + g(x)$

→ Add the functions together.

* Difference between f and g:

$$(f-g)(x) = f(x) - g(x)$$

→ Subtract the functions

* Product of f and g:

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

→ Multiply the functions

* Quotient of f and g:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

→ Divide the functions

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* The domain of combination of functions

Combination	Domain
$f+g$, $f-g$, $f \cdot g$	$D = D_f \cap D_g$ Find domain f, find domain of g and take the intersection
$\frac{f}{g}$ $g \rightarrow$ denominator	$D = D_f \cap D_g$ and exclude the values of x for which $g(x) = 0$

E.g. 1 $f(x) = x - 7$ and $g(x) = x^2 + 2$ (Given)

* Sum: $(f+g)(x) = f(x) + g(x)$

$$= (x-7) + (x^2+2)$$

$$(f+g)(x) = \boxed{x^2 + x - 5}$$

* Difference: $(f-g)(x) = f(x) - g(x)$

$$= (x-7) - (x^2+2)$$

$$= x - 7 - x^2 - 2$$

$$(f-g)(x) = \boxed{-x^2 + x - 9}$$

$$(f\cdot g)(x) = f(x)\cdot g(x)$$
$$= (x-7)(x^2+2)$$
$$= x^3 + 2x - 7x^2 - 14$$
$$(f\cdot g)(x) = \boxed{x^3 - 7x^2 + 2x - 14}$$

To find domain of $f+g$, $f-g$, $f\cdot g$.

We find $\underbrace{D_f}_{\text{domain of } f} \boxed{\cap} \underbrace{D_g}_{\text{domain of } g}$ intersection

$f(x) = x-7$. Domain of f : $D_f = \underbrace{(-\infty, \infty)}_{\text{all real \#s}}$

$g(x) = x^2+2$. Domain of g: $D_g = (-\infty, \infty)$

Intersection: $D_f \cap D_g = (-\infty, \infty)$

So, domain of $f+g$, $f-g$, $f\cdot g$ is $(-\infty, \infty)$

* Quotient:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{x-7}{x^2+2}}$$

To find the domain of $\frac{f}{g}$.

① $D = D_f \cap D_g = (-\infty, \infty)$

② Set $g(x) = 0 \rightarrow x^2 + 2 = 0$

$\rightarrow x^2 = -2 \rightarrow x = \pm\sqrt{-2} = \pm i\sqrt{2}$

$\rightarrow$ non-real solutions $\rightarrow$ we do <u>not</u> need to exclude any real #s from D.

Domain of $\frac{f}{g}$ is $(-\infty, \infty)$.

<u>E.g.</u> $f(x) = x^2 + 1$. $g(x) = x^2 + 2x - 3$.

<u>Q:</u> Find domain of $\frac{f}{g}$.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+1}{x^2+2x-3} = \frac{x^2+1}{(x+3)(x-1)}$$

① Find $D = D_f \cap D_g$.

$D_f = (-\infty, \infty)$; $D_g = (-\infty, \infty)$

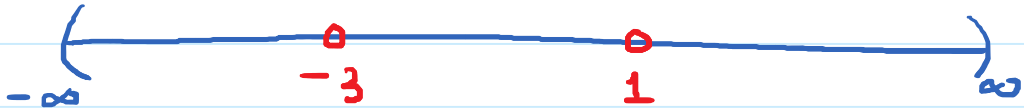
So, $D = (-\infty, \infty)$

② Set $g(x) = 0 \rightarrow x^2 + 2x - 3 = 0$

$\rightarrow (x+3)(x-1) = 0 \rightarrow x = -3 ;\ x = 1$

Exclude these values from D

Conclusion:



Domain of $\frac{f}{g}$: $\boxed{(-\infty,-3)\cup(-3,1)\cup(1,\infty)}$

E.g. $f(x)=\dfrac{3}{x-4}$; $g(x)=\dfrac{1}{x+5}$.

* **Sum:**

$$(f+g)(x)=\underbrace{\frac{3}{x-4}}_{f(x)}+\underbrace{\frac{1}{x+5}}_{g(x)}=\frac{3\cdot(x+5)}{(x-4)\cdot(x+5)}+\frac{1\cdot(x-4)}{(x+5)(x-4)}$$

$$=\frac{3x+15+x-4}{(x-4)(x+5)}$$

$$(f+g)(x)=\boxed{\frac{4x+11}{(x-4)(x+5)}}$$

* **Difference:**

$$(f-g)(x)=\frac{3}{x-4}-\frac{1}{x+5}=\frac{3\cdot(x+5)}{(x-4)\cdot(x+5)}-\frac{1\cdot(x-4)}{(x+5)\cdot(x-4)}$$

$$=\frac{3x+15-(x-4)}{(x-4)(x+5)}=\frac{3x+15-x+4}{(x-4)(x+5)}$$

$$=\boxed{\frac{2x+19}{(x-4)(x+5)}}.$$