Piecewise Defined Functions

Monday, February 18, 2019 1:16 PM

Formula 1 Domain 1

E.g. $f(x) = \begin{cases} 75 & \text{if } 0 \le x \le 60 \\ 75 + 2(x-60) & \text{if } x > 60 \end{cases}$

Formula 2 Domain 2

$f(2) = 75$.

2 belongs to domain 1 → Formula 1

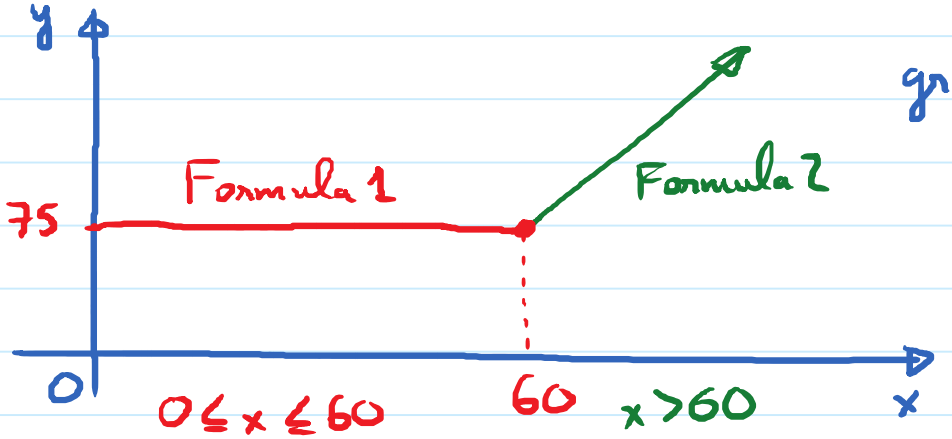
$f(70) = 75 + 2(70-60) = 95$

70 belongs to domain 2 → Formula 2

$f(60) = 75$.

60 Domain 1

Evaluate piecewise function



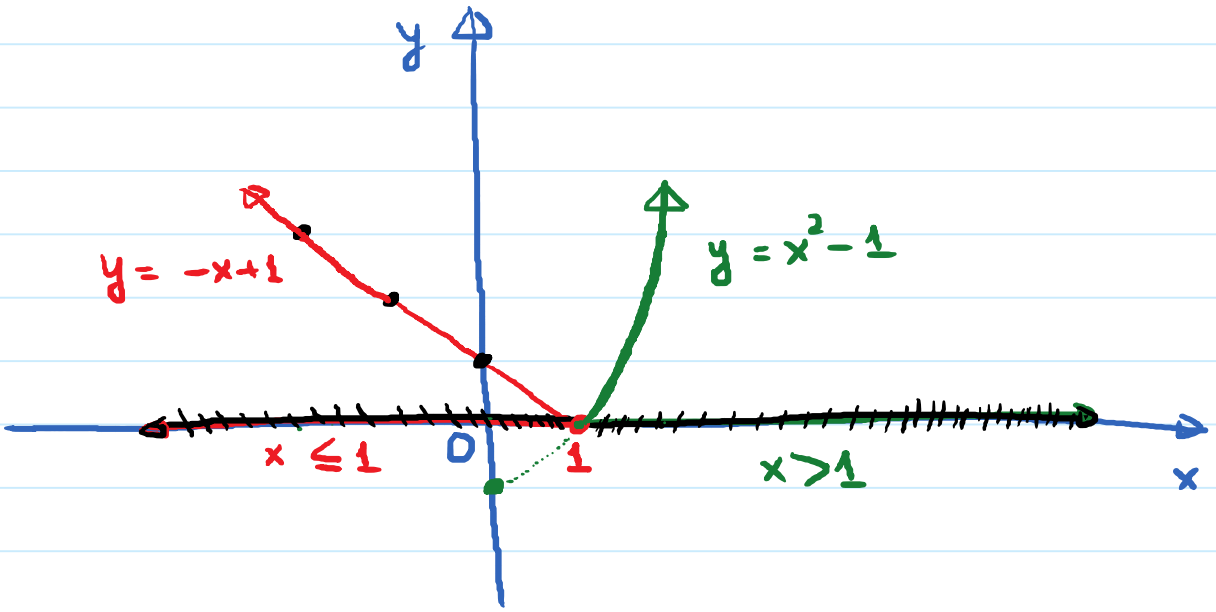
graph of $y = f(x)$

Graph piecewise functions:

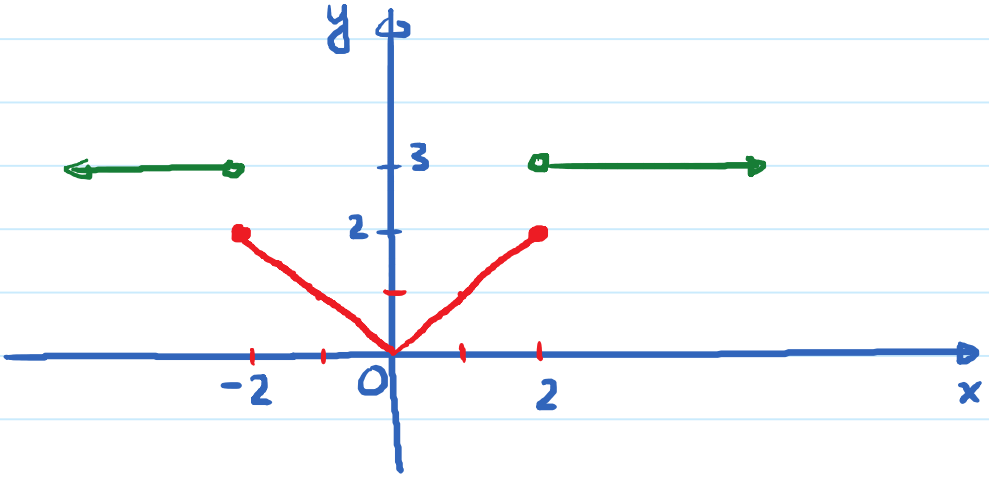
E.g. $f(x) = \begin{cases} -x+1 & \text{if } x \le 1 \\ x^2-1 & \text{if } x > 1 \end{cases}$

$y = -x+1$

$y = x^2 - 1$

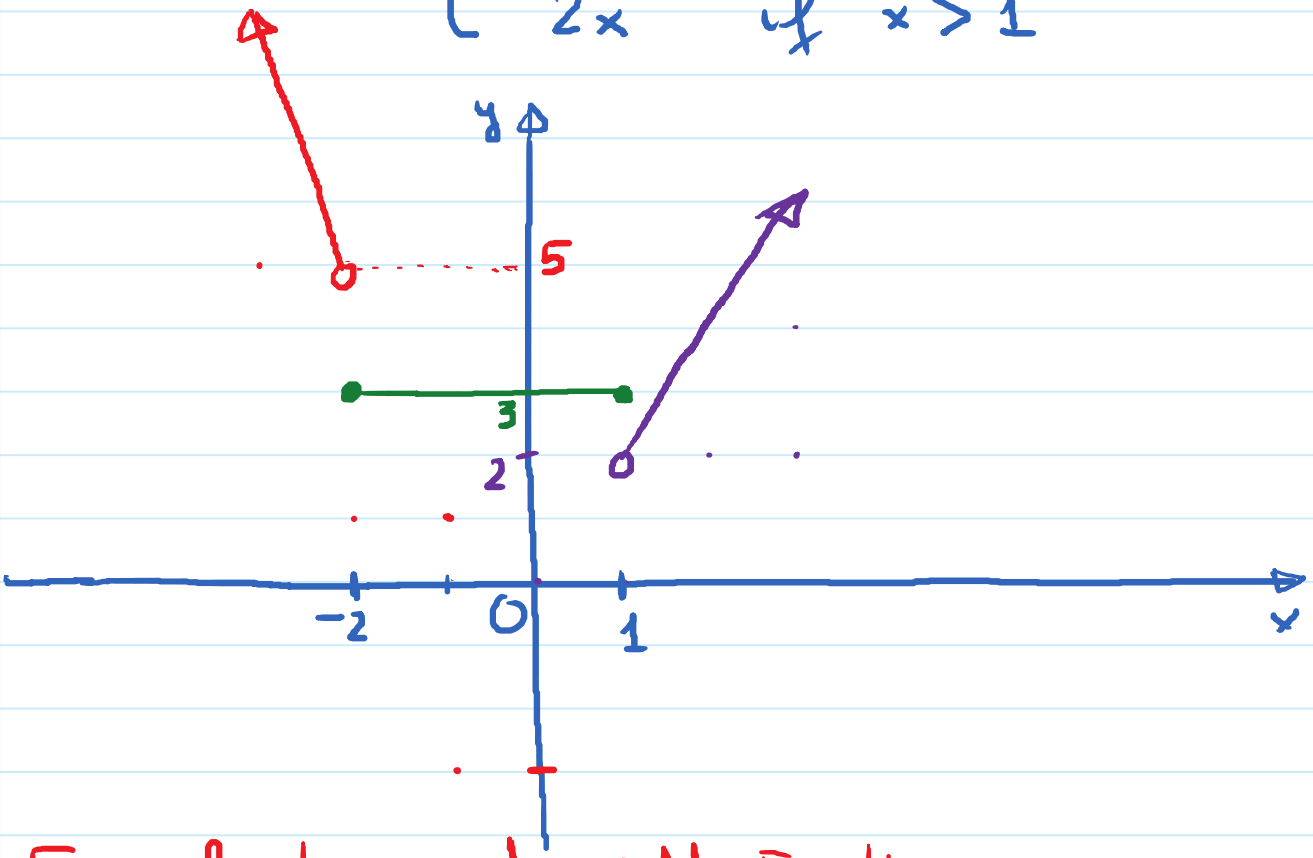


E.g. $g(x) = \begin{cases} |x| & \text{if } -2 \le x \le 2 \\ 3 & \text{, otherwise} \end{cases}$

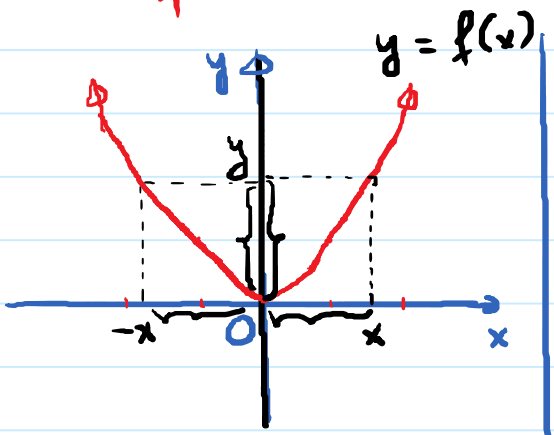


E.g.

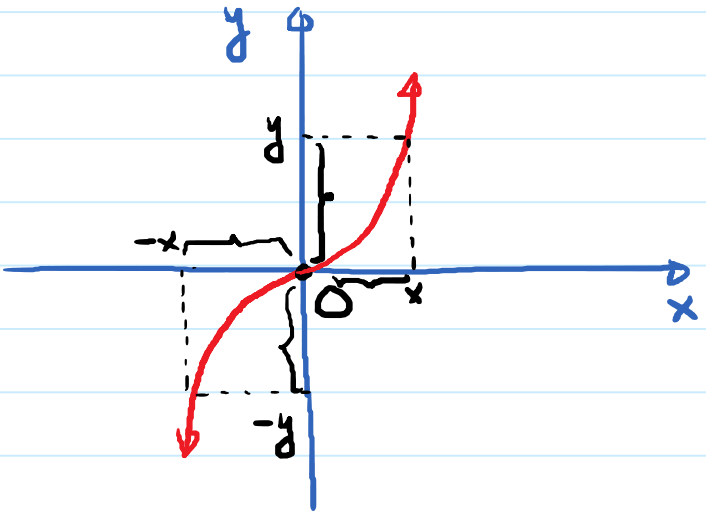
$$f(x) = \begin{cases} -4x - 3 & \text{if } x < -2 \\ 3 & \text{if } -2 \le x \le 1 \\ 2x & \text{if } x > 1 \end{cases}$$



Even functions and Odd Functions



Even: symmetric w.r.t. y-axis



Odd: symmetric w.r.t origin

How to test whether a function is even or odd given the formula of the function $y = f(x)$

$f(-x) = f(x)$ ⟷ f is even

$f(-x) = -f(x)$ ⟷ f is odd

Process: Replace x by $-x$ in the formula for f and simplify.

(a) If the simplified expression is equal to $f(x)$, then f is even.

(b) If the simplified expression is equal to $-f(x)$, then f is odd

(c) If neither, then f is neither odd nor even.

E.g. $f(x) = x^3 + 4x$

Step 1: Replace x by $-x$ and simplify:

$$f(-x) = (-x)^3 + 4(-x)$$

$$= -x^3 - 4x$$

$$= -\underbrace{(x^3 + 4x)}_{f(x)} = -f(x)$$

Step 2: Since $f(-x) = -f(x)$, f is **odd**

E.g. $g(x) = \dfrac{4x^3 + x}{x^7}$

$$g(-x) = \frac{4(-x)^3 + (-x)}{(-x)^7}$$

$$= \frac{-4x^3 - x}{-x^7} = \frac{\cancel{-}(4x^3 + x)}{\cancel{-}x^7}$$

$$= \frac{4x^3 + x}{x^7} = g(x)$$

$g(-x) = g(x) \rightarrow g$ is even

E.g. $h(x) = x^6 - 4x$

$h(-x) = (-x)^6 - 4(-x)$

$= x^6 + 4x$

not the same
not the opposite.

So, h is neither odd nor even.

E.g. $f(x) = x\sqrt{4 - x^8}$

Odd ? Even ? Neither

$f(-x) = -x \cdot \sqrt{4 - (-x)^8}$

$= -x \cdot \sqrt{4 - x^8} = -f(x)$

$\longrightarrow$ f is odd.