Polynomial and Rational Inequalities

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Polynomial Inequalities

1. Graphing Method.

$f(x)$

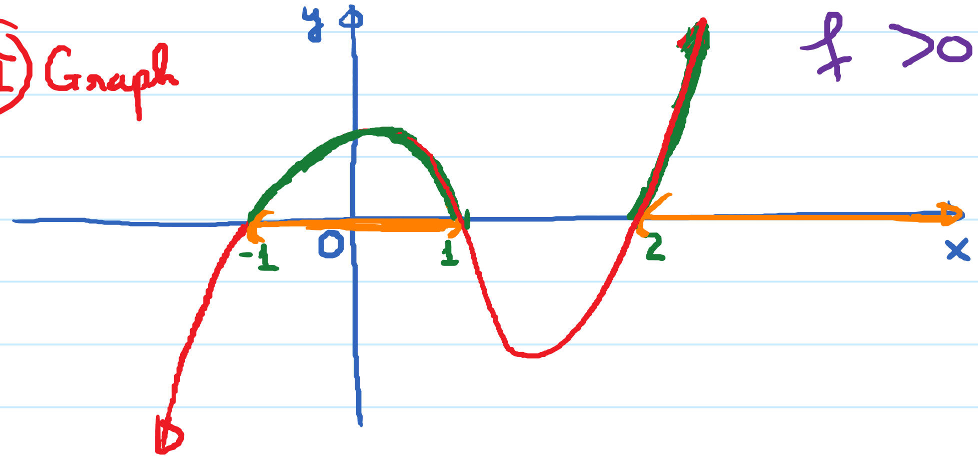
E.g. $\boxed{(x-1)(x+1)(x-2)}\ \boxed{>0}$ → when is $f(x)>0$

I. End Behavior of f: leading term: $1\cdot x^3$

II. Zeros: $1, -1, 2$

Mult. $1, 1, 1$ → cross at zeros.

III. Graph



$f>0$ → graph above x-axis

Solution: $(-1,1)\cup(2,\infty)$

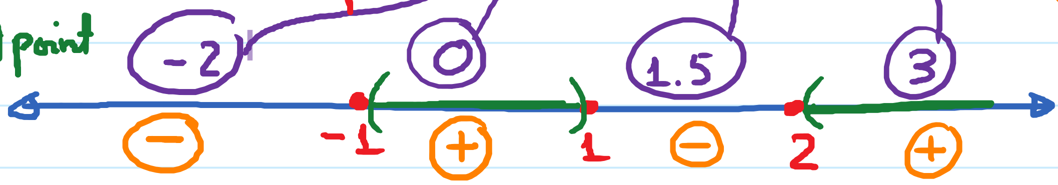
2. Number line method. $f(x) = (x-1)(x+1)(x-2) > 0$

means $\oplus$

I. Zeros: $-1, 1, 2$.

II. Number and Test points

Test point



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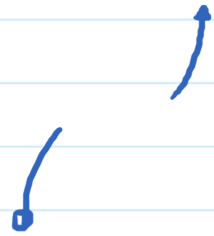
Solution: $\boxed{(-1,1) \cup (2,\infty)}$

Ex. Use either method to solve the inequality:

(a) $(x+1)^2(x+2)(x+3)^2 \leq 0$

(b) $(x+1)^2(x+2)(x+3)^2 \geq 0$

(a) leading term $= x^5$. End Behavior:

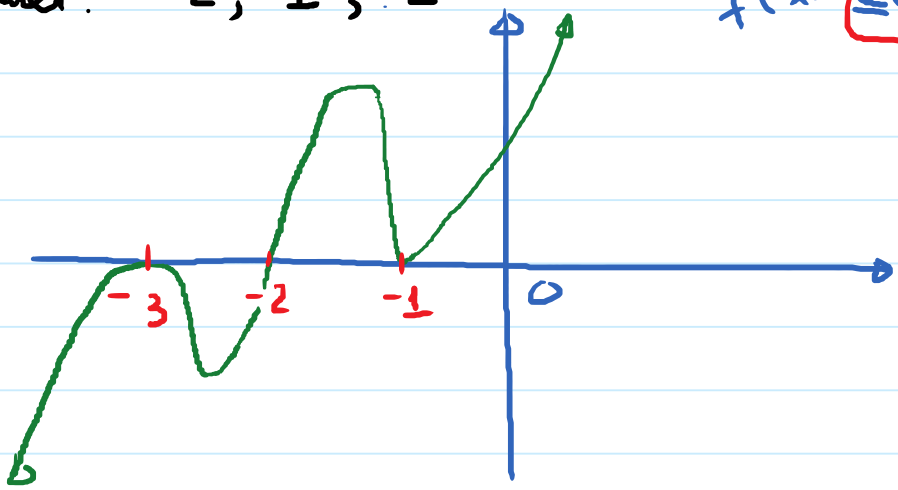


Zeros: $-1, -2, -3$.

Mult: 2, 1, 2

$f(x) \boxed{\leq 0}$

below & including 0



Solution: $(-\infty, -3] \cup [-3, -2] = (-\infty, -2]$

(b) $f(x) \geq 0$ → above, including 0

Solution: $[-2, -1] \cup [-1, \infty)$

$= [-2, \infty)$

Rational Inequalities:

Number line method:

E.g. Solve the inequality: $\boxed{\dfrac{x-1}{x-4}} > 0$.

$f(x)$

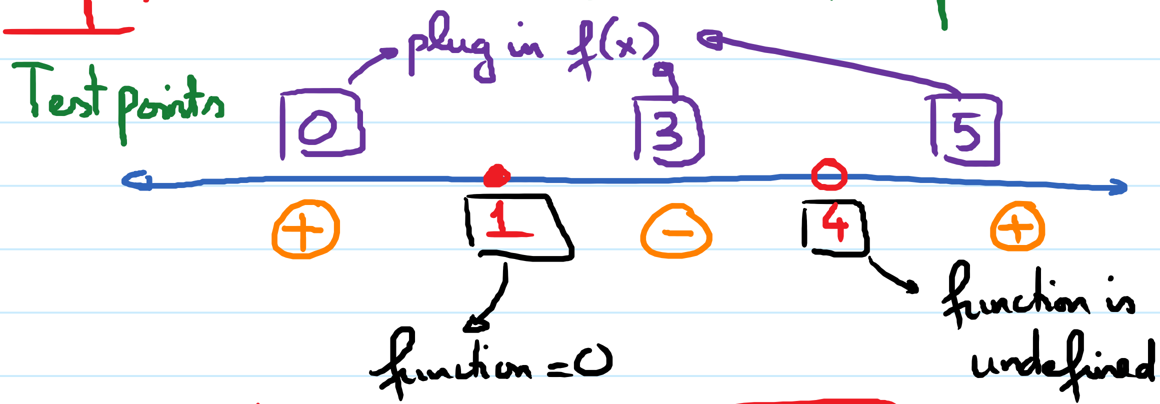
> 0 means $+$

Step 1: Set num $= 0$ and denom $= 0$.

$x - 1 = 0 \rightarrow x = 1$

$x - 4 = 0 \rightarrow x = 4$.

Step 2: Use number line and test points.



Solution: $\boxed{(-\infty, 1) \cup (4, \infty)}$

Note: if we solve $f(x) \geq 0$, then solution is: $(-\infty, 1] \cup (\boxed{4}, \infty)$

the # 4 makes function undefined.

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Ex. Solve the inequality.

ⓐ $\boxed{\dfrac{2x+3}{x+4}} \geq 0$ ⓑ $\dfrac{x-2}{x+6} < 1$.

Sol: $f(x)$

ⓐ Step 1: $2x+3=0$; $x+4=0$

$\rightarrow x = -\dfrac{3}{2}$; $x=-4$

Step 2: Plug into $f(x)$

Test points $\boxed{-6}$ $\boxed{-3}$ $\boxed{0}$

$+$ $\quad -4 \quad$ $-$ $\quad -\dfrac{3}{2} \quad$ $+$

$f(x) \boxed{\geq 0}$ → positive or $=0$

Solution: $\boxed{(-\infty, -4) \cup \left[-\dfrac{3}{2}, \infty\right)}$

ⓑ $\dfrac{x-2}{x+6} < 1.$ → not zero → Subtract 1 from both sides.

$$\frac{x-2}{x+6} - \frac{1\cdot(x+6)}{1\cdot(x+6)} < 0$$

$$\frac{x-2}{x+6} - \frac{x+6}{x+6} < 0$$

$$\frac{x-2-(x+6)}{x+6} < 0$$

$$\frac{x-2-x-6}{x+6} < 0$$

$$\frac{-8}{x+6} < 0$$

want $-$

Set $x+6=0 \rightarrow x=-6$

Test points: -7, 0

-7: $+$ | -6 | 0: $-$

Solution: $\boxed{(-6, \infty)}$

Note: If RHS is not zero, we have to subtract both sides by a quantity to make RHS $= 0$ and combine fractions on LHS before solving