Properties of Logarithms and Log, Exp Equations.

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3 Important Properties of Log Function:

I Product Rule.

$$\boxed{\log_a(M) + \log_a(N) = \log_a(MN)}$$

E.g.

Given : $\log_a 2 = 0.301$ and $\log_a 5 = 0.699$.

Find : $\log_a(10)$

By product rule: $\underbrace{\log_a(\overset{M}{2})}_{0.301} + \underbrace{\log_a(\overset{N}{5})}_{0.699} \overset{\text{Product Rule}}{=} \log_a(\overset{M \cdot N}{10})$

$0.301 + 0.699 = 1$

So, $\log_a(10) = 1$

E.g. $\log_3(\underbrace{x+1}_{M}) + \log_3(\underbrace{x}_{N}) \overset{\text{Product Rule}}{=} \log_3[\overbrace{(x+1)\cdot x}^{M \cdot N}]$

Note: If base $a = e$, then the product rule becomes:

$$\boxed{\ln(M) + \ln(N) = \ln(MN)}$$

II Quotient Rule

$$\boxed{\log_a(M) - \log_a(N) = \log_a\left(\frac{M}{N}\right)}$$

E.g. $\log_b A = 2.471$; $\log_b B = 0.19$ (Given)

Find $\log_b\left(\frac{A}{B}\right)$

Quotient Rule

$$\underbrace{\log_b A}_{2.471} - \underbrace{\log_b B}_{0.19} = \log_b\left(\frac{A}{B}\right)$$

$$2.471 - 0.19 = 2.281$$

So, $\log_b \frac{A}{B} = 2.281$.

E.g. $\log(\underbrace{x^2+1}_{M}) - \log(\underbrace{x^2+7}_{N}) = \log\left(\boxed{\frac{x^2+1}{x^2+7}}\right)$

$\frac{M}{N}$

Quotient rule

Note: If base $a = e$, then :

$$\boxed{\ln(M) - \ln(N) = \ln\left(\frac{M}{N}\right)}$$

III Power Rule:

$$\boxed{\log_a(M^{\boxed{p}}) = p \cdot \log_a(M)}$$

E.g. $\log_a(2) = 0.31$. (Given)

Find $\log_a(8)$?

Power Rule

$$\log_a(8) = \log_a(2^{\boxed{3}}) = 3 \cdot \log_a(2)$$

$$= 3 \cdot (0.31) = \boxed{0.93}.$$

E.g. $\log_2((x+1)^{\boxed{3}}) = 3 \cdot \log_2(x+1)$

Power Rule.

Note: If base $a = e$, then:

$$\boxed{\ln(M^p) = p \cdot \ln(M)}$$

Expand log expressions using these properties.

E.g. $\log_3(x^3y)$. Expand.

By product rule:

$$\log_3(\overset{M}{\boxed{x^3}}\cdot\overset{N}{\boxed{y}}) = \log_3(\overset{M}{\boxed{x^3}}) + \log_3(\overset{N}{\boxed{y}})$$

Power Rule

$$= \boxed{3\log_3(x) + \log_3(y)}$$

So,

$$\log_3(x^3y) = 3\log_3(x) + \log_3(y)$$

E.g. $\log_2(x^4y^{\frac{1}{2}}z^{10})$. Expand

$$\log_2(x^4y^{\frac{1}{2}}z^{10}) = 4\log_2(x) + \frac{1}{2}\log_2(y) + 10\log_2(z)$$

Product and Power Rule

E.g. $\ln\left(\frac{x^2}{y^5}\right)$. Expand

By Quotient Rule: $\ln\left(\frac{x^2}{y^5}\right) = \ln(x^2) - \ln(y^5)$

Power Rule

$$= 2\ln x - 5\ln y.$$

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E.g. $\log\left(\frac{x^3 y^2}{z^4}\right)$

$= 3\log x + 2\log y - 4\log z$

Quotient and Power Rule

Condense Log Expressions using Properties.

E.g. $\boxed{2}\log_3 x + \boxed{3}\log_3 y$. Condense.

$= \log_3 x^2 + \log_3 y^3$

Power Rule

$= \log_3(x^2 y^3)$

Product Rule.

E.g. $7\log_2 x + \frac{1}{2}\log_2 y + \frac{1}{3}\log_2 z$.

$= \log_2\left(x^7 y^{\frac{1}{2}} z^{\frac{1}{3}}\right) = \log_2\left(x^7 \sqrt{y}\sqrt[3]{z}\right)$

Power and Product Rule

rewrite $y^{\frac{1}{2}} = \sqrt{y}$

$z^{\frac{1}{3}} = \sqrt[3]{z}$.

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E.g. $2\ln x - 3\ln y$. Condense.

$$= \ln\left(\frac{x^2}{y^3}\right)$$

Product, Quotient Rule

E.g. $\log_5(x+1) - 2\log_5(x-1)$

Power $= \log_5(x+1) - \log_5(x-1)^2$

$$= \boxed{\log_5\left(\frac{x+1}{(x-1)^2}\right)}$$