Quadratic Equations

Monday, January 28, 2019 12:57 PM

I Factoring

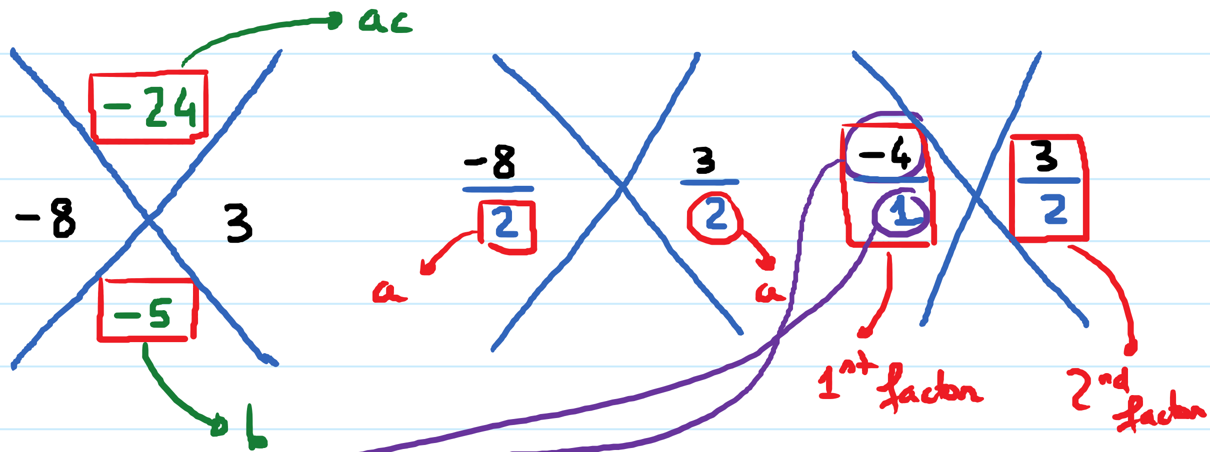
Standard form of a quadratic equation:

$$\boxed{ax^2 + bx + c = 0}$$

a, b, c: constants ; $a \neq 0$.

E.g. $\boxed{2}x^2 - 5x - 12 = 0$. Solve this equation by factoring.

a b c



$\rightarrow (1x - 4)(2x + 3) = 0$

$\rightarrow (x - 4)\cdot(2x + 3) = 0$

$x - 4 = 0$ $\qquad$ $2x + 3 = 0$

$\boxed{x = 4}$ $\qquad$ $\boxed{x = -\frac{3}{2}}$

Solution set : $\boxed{\left\{ -\frac{3}{2}, 4 \right\}}$

Difference between Squares

$$\boxed{A^2 - B^2 = (A+B)(A-B)}$$

E.g. $6x^2 - 54 = 0$

$6(x^2 - 9) = 0$

$6 \cdot (x-3)(x+3) = 0$

$x - 3 = 0$; $x + 3 = 0$

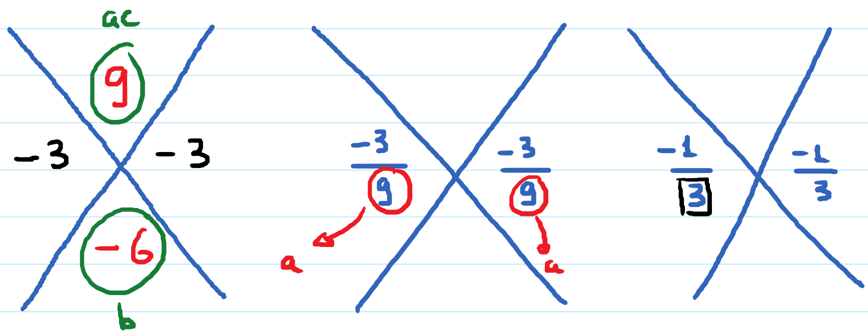
$\boxed{x = 3}$; $\boxed{x = -3}$ Solution set: $\{3, -3\}$

E.g. $9x^2 - 4x - 4 = 2x - 5$

$\boxed{9x^2 - 6x + 1} = 0$

$(3x)^2 - 2 \cdot 3x \cdot 1 + (1)^2$

$A^2 - 2 \cdot A \cdot B + B^2$



$(3x - 1)(3x - 1) = 0 \longrightarrow (\underbrace{3x}_{A} - \underbrace{1}_{B})^2$

$3x - 1 = 0$

$\boxed{x = \frac{1}{3}}$

Square of a difference : $\boxed{(A-B)^2 = A^2 - 2AB + B^2}$

Square of a sum : $\boxed{(A+B)^2 = A^2 + 2AB + B^2}$

II) Extraction of Roots :

E.g. $7x^2 = 4 \longleftrightarrow x^2 = \frac{4}{7}$

$\longleftrightarrow x = \pm\sqrt{\frac{4}{7}} \longleftrightarrow x = \pm \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$\longleftrightarrow \boxed{x = \pm \frac{2\sqrt{7}}{7}}$

Solution set : $\left\{ \frac{2\sqrt{7}}{7}, -\frac{2\sqrt{7}}{7} \right\}$

$$\boxed{\begin{gathered}(\text{Stuff})^2 = a \\ \longrightarrow \text{Stuff} = \pm\sqrt{a}\end{gathered}}$$

Extraction of Roots

Monday, January 28, 2019 1:27 PM

E.g. $(2x-1)^2 = 12$

$2x - 1 = \pm\sqrt{12}$ → 4.3

$2x - 1 = \pm 2\sqrt{3}$

$2x = 1 \pm 2\sqrt{3}$

$$\boxed{x = \frac{1 \pm 2\sqrt{3}}{2}}$$

Solution set : $\left\{ \frac{1+2\sqrt{3}}{2}, \frac{1-2\sqrt{3}}{2} \right\}$

Monday, January 28, 2019 1:30 PM

$\boxed{i^2 = -1}$ → imaginary unit.

E.g. $(5x+3)^2 = -4$

$5x + 3 = \pm\sqrt{-4} = \pm\sqrt{4i^2}$

$5x + 3 = \pm 2i$

$5x = -3 \pm 2i$

$$\boxed{x = \frac{-3 \pm 2i}{5}}$$

Solution set : $\underbrace{\left\{ \frac{-3+2i}{5}, \frac{-3-2i}{5} \right\}}_{\text{Non-real solutions.}}$