E.g. $2(2x+1)^2 + 3 = 11$

$2(2x+1)^2 = 8$

$(2x+1)^2 = 4$

$2x+1 = \pm 2$

$2x+1 = 2$ or $2x+1 = -2$

$2x = 1$; $\boxed{x = \frac{1}{2}}$ $2x = -3$; $\boxed{x = -\frac{3}{2}}$

Solution set: $\left\{\frac{1}{2}, -\frac{3}{2}\right\}$

(III) Completing the Square:

E.g. Solve the quadratic equation by completing the square.

$2x^2 + 14x + 4 = 0$

Step 1: Divide both sides by a.

$x^2 + 7x + 2 = 0$

Step 2: Move c to the other side.

$x^2 + 7x = -2$

Step 3: Add the square of half of b to both sides:

$b = 7 \rightarrow$ half of $b = \frac{7}{2} \rightarrow$ square of that $\frac{49}{4}$

$$x^2 + 7x + \frac{49}{4} = -2 + \frac{49}{4}$$

Step 4: The left hand side = square of a sum or square of a difference.

$$\left(x + \frac{7}{2}\right)^2 = \frac{-2 \cdot 4}{1 \cdot 4} + \frac{49}{4} = \frac{-8 + 49}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{41}{4}$$

Step 5: Extraction of Roots:

$$x + \frac{7}{2} = \pm\sqrt{\frac{41}{4}}$$

$$x + \frac{7}{2} = \pm\frac{\sqrt{41}}{2}$$

$$x = -\frac{7}{2} \pm \frac{\sqrt{41}}{2}$$

$$x = \frac{-7 \pm \sqrt{41}}{2}$$

E.g. Solve by completing the square:

$$x^2 - 6x + 7 = 0$$

Step 2: $x^2 - 6x = -7$ (Move c to the other side)

Step 3: $x^2 - 6x + 9 = -7 + 9$ (Add $\left(\frac{b}{2}\right)^2$ to both sides)

Step 4: $(x-3)^2 = 2$ (Get perfect square on left hand side)

Step 5: $x - 3 = \pm\sqrt{2}$ (Extraction of root)

$$\boxed{x = 3 \pm \sqrt{2}}$$

(IV) Using Quadratic Formula:

The solutions to $ax^2 + bx + c = 0$ are given by the formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

E.g. Solve : $x^2 - 4x + 1 = 0$ by quadratic formula.

$a = 1$; $b = -4$; $c = 1$

Quadratic formula: $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$x = \dfrac{4 \pm \sqrt{12}}{2}$ $\left(\sqrt{12} = \sqrt{3 \cdot 4} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}\right)$

$x = \dfrac{4 \pm 2\sqrt{3}}{2} \rightarrow x = \dfrac{2(2 \pm \sqrt{3})}{2}$

$$\boxed{x = 2 \pm \sqrt{3}}$$

E.g. $(3x - 5)(3x - 3) = -1$. → Solve.

$9x^2 - 9x - 15x + 15 = -1$

$9x^2 - 24x + 16 = 0$

$a = 9$; $b = -24$; $c = 16$

$x = \dfrac{24 \pm \sqrt{(-24)^2 - 4 \cdot 9 \cdot 16}}{2 \cdot 9}$

$x = \dfrac{24 \pm \sqrt{576 - 576}}{18} = \dfrac{24}{18} = \boxed{\dfrac{4}{3}}$

Where does the quadratic formula come from?

$$ax^2 + bx + c = 0$$

Step 1: Divide both sides by a.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step 2:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 2: Add $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c \cdot 4a}{a \cdot 4a} + \frac{b^2}{4a^2}$$

Step 3:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 4:

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$