How do we obtain the standard form from the general form?

Given general form: $f(x) = ax^2 + bx + c$.

Want: standard form: $f(x) = a(x-h)^2 + k$

Vertex formula:

$$\boxed{\begin{aligned} x_{vertex} &= h = -\frac{b}{2a} \\ y_{vertex} &= k = f(h) = f\left(-\frac{b}{2a}\right) \end{aligned}}$$

E.g. $f(x) = -x^2 - 4x - 3$. ← general form.

Q1: Rewrite f in standard form. Identify vertex, axis of symmetry, up or down?

Q2: Find 2 pairs of points on both sides of axis of symmetry and graph.

Sol:

Q1: Standard form: $f(x) = a(x-h)^2 + k$

$$a = -1; \quad h = -\frac{b}{2a} = -\frac{(-4)}{2(-1)} = -2$$

$K = f(-2) = -(-2)^2 - 4(-2) - 3$

$k = 1$

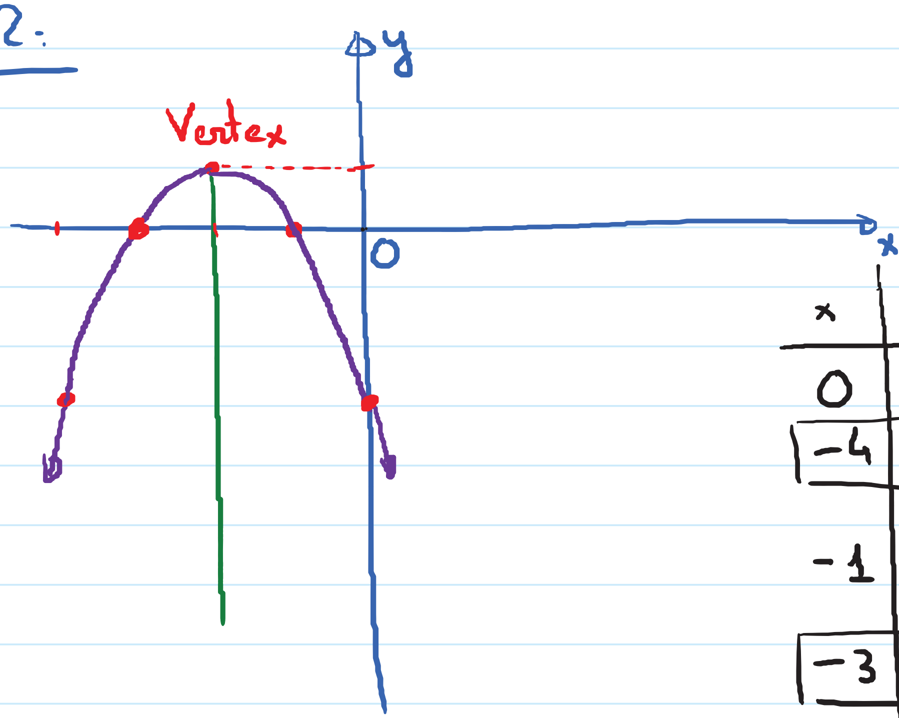
Standard form: $\boxed{f(x) = -(x+2)^2 + 1}$.

Vertex : $(h, k) = (-2, 1)$

Axis of symmetry : $x = -2$

Open down.

Q2:



x	$y = f(x)$	
0	-3	
$\boxed{-4}$	$\boxed{-3}$	free by symmetry
-1	0	
$\boxed{-3}$	$\boxed{0}$	free by symmetry

E.x. Given $f(x) = 2x^2 - 10x + 8$

Q1: Rewrite in standard form. Find vertex, etc.

Q2: Graph. (find 2 pairs of symmetric points on both sides of axis of symmetry)

Sol:

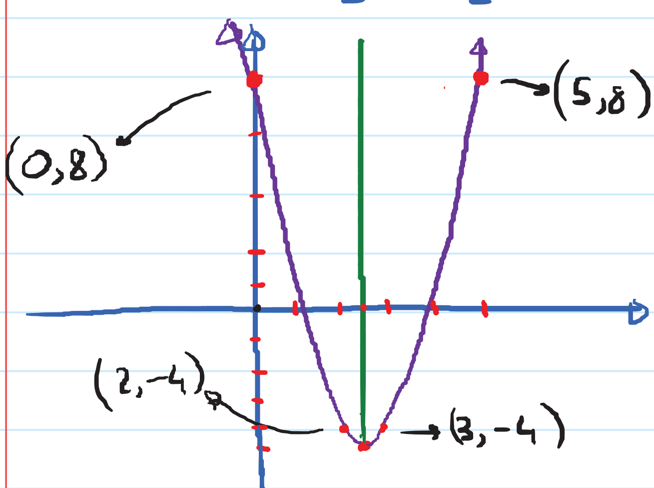
Q1: Standard form:

$$f(x) = 2\left(x - \frac{5}{2}\right)^2 - \frac{9}{2}$$

$$h = -\frac{b}{2a} = -\frac{(-10)}{4} = \frac{5}{2}.$$

$$k = f\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right)^2 - 10\cdot\frac{5}{2} + 8 = -\frac{9}{2}$$

Vertex: $\left(\frac{5}{2}, -\frac{9}{2}\right)$, A.O.S: $x = \frac{5}{2}$; opens up.

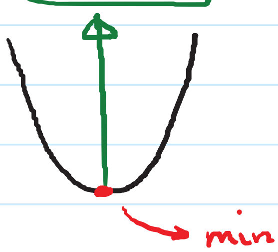


x	y = f(x)
0	8
5	8
2	-4
3	-4

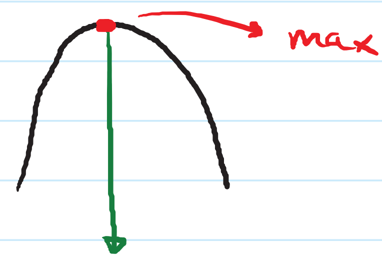
Range, Max / Min of quadratic functions.

Given a quadratic function : $f(x) = ax^2 + bx + c$

$\boxed{a > 0}$



$\boxed{a < 0}$



$a > 0$	$a < 0$
Min $= (h, k)$	Max $= (h, k)$
Min Value $= k$	Max Value $= k$
Range $= [k, \infty)$	Range $= (-\infty, k]$

$$\boxed{h = -\frac{b}{2a} \; ; \quad k = f(h)}$$

E.g. $f(x) = 2x^2 - 20x - 4$

(a) $a = 2 > 0 \longrightarrow f$ has a minimum

(b) $h = -\frac{b}{2a} = \frac{20}{4} = 5$; $k = f(5) = -54$

Min value $= -54$. It occurs at $x = 5$

(c) Domain: $(-\infty, \infty)$

Range: $[-54, \infty)$

An application

HW #20