# Rational Functions

Wednesday, March 27, 2019 1:04 PM

A rational function is a function of the form:

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials.

E.g. $f(x) = \dfrac{2x+3}{5x+7}$ ; $g(x) = \dfrac{7}{x^2-5}$

$$h(x) = \frac{x^2+1}{x^3-x^2+x-1}$$

## Domain of a Rational Function:

Key: To find the domain of $f(x) = \dfrac{p(x)}{q(x)}$

Step 1: Set Denom $q(x) = 0$. Solve for $x$.

Step 2: Domain = set of all real numbers except for the values in Step 1.

E.g. Find the domain

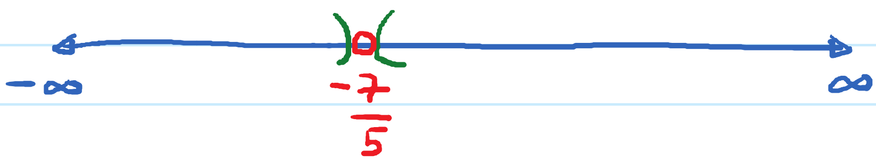
(a) $f(x) = \dfrac{2x+3}{5x+7}$

(1) $5x+7 = 0$ (Set Denom $= 0$)

$x = -\frac{7}{5}$

(2) Domain = all real #s except for $x = -\frac{7}{5}$

Interval notation: $\boxed{(-\infty, -\frac{7}{5}) \cup (-\frac{7}{5}, \infty)}$




Set Builder notation: $\boxed{\{x \mid x \neq -\frac{7}{5}\}}$

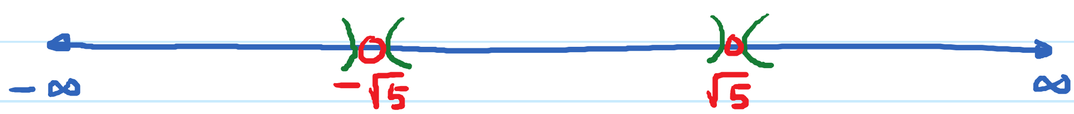
(b) $g(x) = \frac{7}{x^2 - 5}$

(1) $x^2 - 5 = 0$ (Set Denom = 0)

$\rightarrow x^2 = 5 \rightarrow x = \pm\sqrt{5}$.

(2) Domain in interval notation:

$\boxed{D = (-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)}$




Set Builder Notation: $\boxed{\{x \mid x \neq \sqrt{5} \text{ and } x \neq -\sqrt{5}\}}$

c) $h(x) = \dfrac{x^2+1}{x^3-x^2+x-1}$

1) $\underbrace{x^3-x^2}+x-1=0$ (Set Denom. $=0$)

$x^2(x-1)+(x-1)=0$ (Factor by grouping)

$(x-1)(x^2+1)=0$

$x-1=0$ or $x^2+1=0$

$\rightarrow x=1$ $\quad$ $x^2=-1 \rightarrow x=\pm\sqrt{-1}$

Real

$\rightarrow x=\pm i$

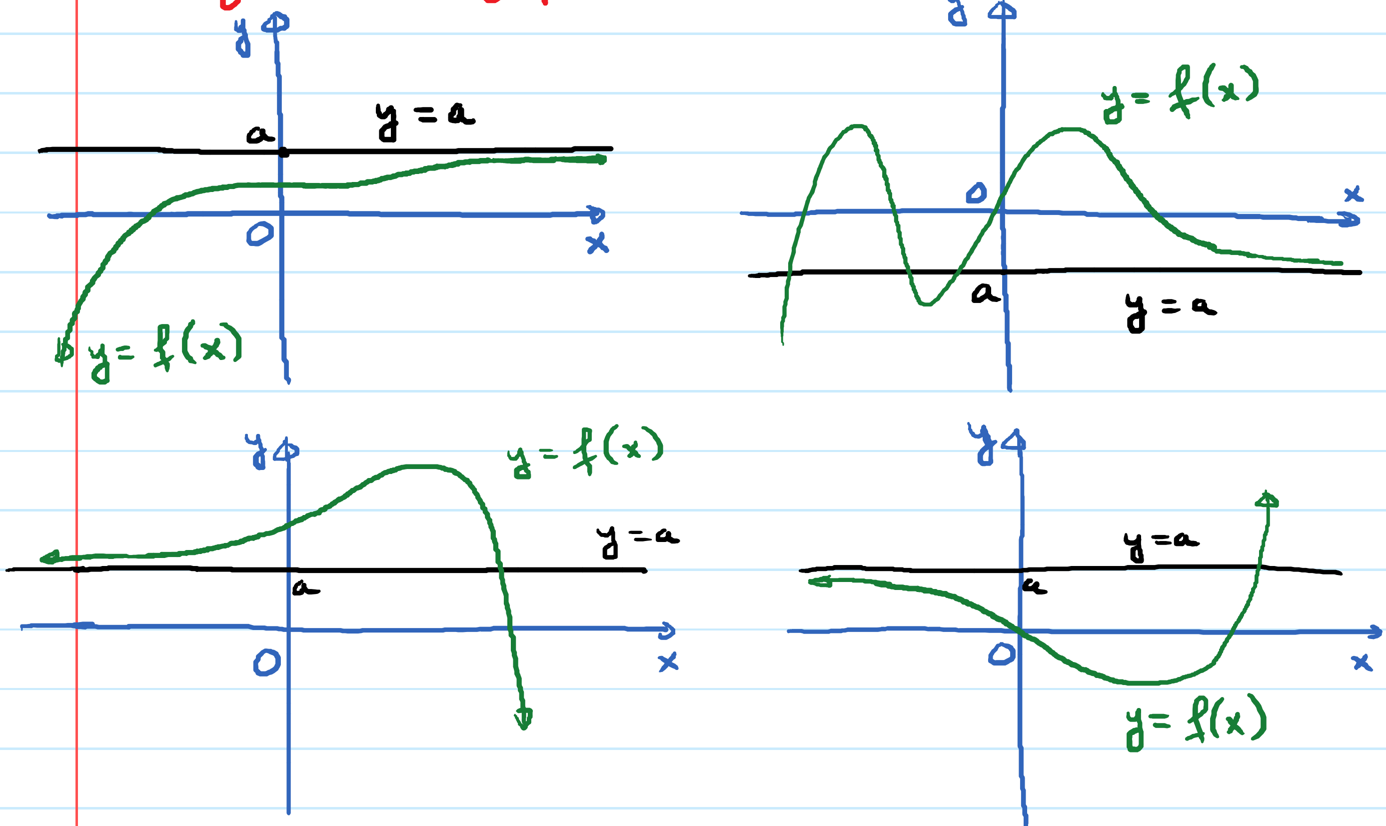
Non real

2) Domain = all real #s except for 1

$= \boxed{(-\infty,1)\cup(1,\infty)}$

or $\boxed{D=\{x \mid x \neq 1\}}$

# Horizontal Asymptotes:




Process for find the Horizontal Asymptotes (H. A.) of a rational function $f(x) = \dfrac{p(x)}{q(x)}$

Case 1: $\boxed{\text{Degree of } p(x) = \text{Degree of } q(x)}$

H. A. is : $\boxed{y = \dfrac{a}{b}}$

where $a$ = leading coefficient of $p(x)$.

$b$ = leading coefficient of $q(x)$.

E.g. $f(x) = \dfrac{\boxed{2}x^2+5}{\boxed{7}x^2+3}$

Degree of top = 2 = Degree of bottom.

So, $\boxed{\text{HA: } y = \dfrac{2}{7}}$

Case 2: $\boxed{\text{Degree of } p(x) < \text{Degree of } q(x)}$

H.A. $\boxed{y = 0}$

E.g. $f(x) = \dfrac{2x^2+5}{x^3+3x^2+x+1}$

Degree of top = 2 < 3 = degree of bottom.

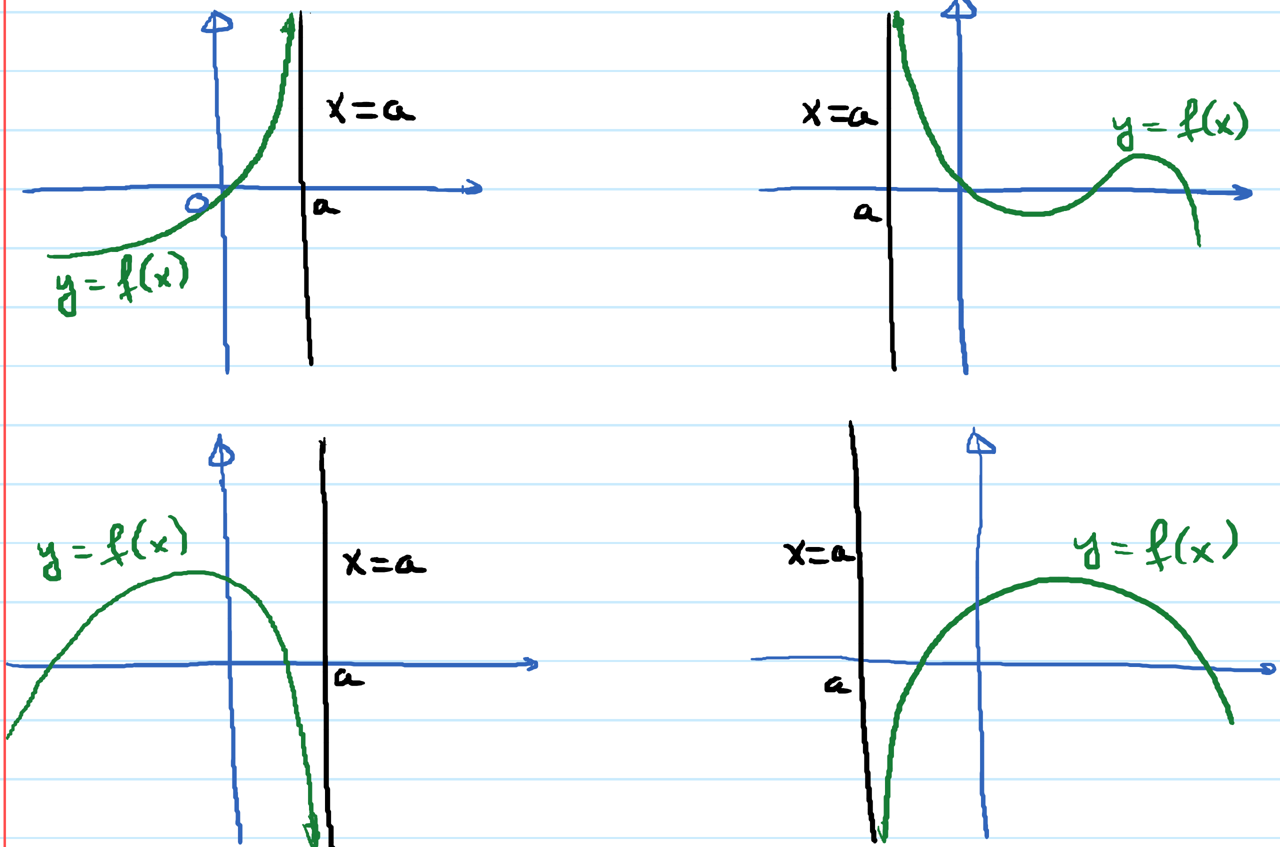
So, HA : $\boxed{y = 0}$

Case 3: $\boxed{\text{Degree of } p(x) > \text{Degree of } q(x)}$

HA : There is NONE.

E.g. $f(x) = \dfrac{x^4+x^2-1}{2x^3+7} \longrightarrow$ No H.A.

# Vertical Asymptotes:




## Process for finding Vertical Asymptotes (V.A.) of a Rational Function.

E.g. $f(x) = \dfrac{x^2 - 1}{x^2 - 3x + 2}$

Step 1: Factor num. and denom. completely.

$$f(x) = \frac{(x+1)(x-1)}{(x-2)(x-1)}$$