E.g. Given $f(x) = -2x^2 + 1$.

Find $f(-3)$, $f(0)$, $f(2)$, $f(x-1)$

E.g. Given $g(x) = x^2 - 2x + 3$.

Find $g(-1)$, $g(0)$, $g(2)$, $g(2x)$, $g(x-1)$

Solution:

1. $f(-3) = -2\cdot(-3)^2 + 1 = \boxed{-17}$

$f(0) = -2\cdot(0)^2 + 1 = \boxed{1}$

$f(2) = -2\cdot(2)^2 + 1 = \boxed{-7}$

$$f(x-1) = -2(x-1)^2 + 1$$
$$= -2\,\underbrace{(x-1)\cdot(x-1)} + 1$$
$$= -2\left(x^2 - 2x + 1\right) + 1$$
$$= -2x^2 + 4x - 2 + 1 = \boxed{-2x^2 + 4x - 1}$$

2 $g(x) = x^2 - 2x + 3$

$g(-1) = \boxed{6}$

$g(0) = \boxed{3}$

$g(2) = \boxed{3}$

$$g(2x) = (2x)^2 - 2(2x) + 3.$$

$$= \boxed{4x^2 - 4x + 3}$$

$$g(x-1) = (x-1)^2 - 2(x-1) + 3$$

$$= x^2 - 2x + 1 - 2x + 2 + 3$$

$$= \boxed{x^2 - 4x + 6}$$

E.g.

$$w(x) = \frac{3x}{x-1}$$

$$w(x-1) = \frac{3(x-1)}{x-1-1} = \frac{3x-3}{x-2}$$

$$w\left(\frac{1}{x}\right) = \frac{3\left(\frac{1}{x}\right)}{\frac{1}{x} - \frac{1 \cdot x}{1 \cdot x}} = \frac{\frac{3}{x}}{\frac{1-x}{x}}$$

$$= \frac{3}{x} \cdot \frac{x}{1-x} = \frac{3\cancel{x}}{\cancel{x}(1-x)} = \boxed{\frac{3}{1-x}}$$

The difference quotient:

Given a function $y = f(x)$, the difference quotient is given by the formula:

$$\boxed{\frac{f(x+h) - f(x)}{h}}$$

E.g. Given $f(x) = -3x^2 - 4$. Find the difference quotient.

Step 1: Find $f(x+h) = -3(x+h)^2 - 4$.

$$= -3(x+h)\cdot(x+h) - 4$$

$$= -3(x^2 + 2xh + h^2) - 4$$

$$f(x+h) = \boxed{-3x^2 - 6xh - 3h^2 - 4}$$

Step 2: Plug in the difference quotient formula and simplify:

$$\frac{f(x+h) - f(x)}{h} = \frac{-3x^2 - 6xh - 3h^2 - 4 - (\overbrace{-3x^2 - 4}^{f(x)})}{h}$$

$$= \frac{\cancel{-3x^2} - 6xh - 3h^2 \cancel{-4} + \cancel{3x^2} \cancel{+4}}{h}$$

$$= \frac{-6xh - 3h^2}{h} = \frac{\cancel{h}(-6x - 3h)}{\cancel{h}}$$

$$= \boxed{-6x - 3h} \longrightarrow \text{difference quotient}$$

HW next time: watch second video

and do the HW for this topic.