

## Synthetic Division.

Monday, March 18, 2019 1:10 PM

Idea:  $f(x) = x^3 + 5x^2 + 9x - 7$

$$g(x) = x + 5.$$

Want:  $f(x) \div g(x)$  (or  $\frac{f(x)}{g(x)}$ )

Result of dividing  $f(x)$  by  $g(x)$  consists of the quotient  $q(x)$  and the remainder  $r(x)$

$$\underbrace{f(x)}_{\text{Divisor}} = \underbrace{g(x)}_{\text{Dividend}} \cdot \underbrace{q(x)}_{\text{quotient}} + \underbrace{r(x)}_{\text{remainder}}$$

$$\text{or } \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

Q: How do we find the quotient  $q(x)$  and the remainder  $r(x)$  when we are given  $f(x)$  and  $g(x)$ ?

A: Synthetic Division.

Note: Synthetic Division only works if  $g(x)$  has the form  $g(x) = x - c$  or  $g(x) = x + c$ .

## Process of Synthetic Division.

E.g. Find quotient and remainder for the division:

$$\begin{array}{r} 4x^3 + 3x^2 - 10x + 11 \\ x - 3 \end{array}$$

Step 1:

$$3 \mid 4 \quad 3 \quad -10 \quad 11$$

Step 2:

The diagram illustrates the synthetic division process for  $4x^3 + 3x^2 - 10x + 11$  divided by  $x - 3$ . The divisor  $3$  is written to the left of the dividend coefficients  $4, 3, -10, 11$ . The process involves multiplying the divisor by each coefficient and adding the result to the next coefficient. The results are written below the original coefficients, and the final result is written to the right of the last coefficient.

Step 2 details:

- Initial values:  $4, 3, -10, 11$
- Step 1:  $3 \times 4 = 12$  (labeled "mult."),  $3 + 12 = 15$  (labeled "add").
- Step 2:  $3 \times 15 = 45$  (labeled "mult."),  $-10 + 45 = 35$  (labeled "add").
- Step 3:  $3 \times 35 = 105$  (labeled "mult."),  $11 + 105 = 116$  (labeled "add").
- Final result:  $116$  (boxed, labeled "Remainder").

The quotient coefficients are  $4, 15, 35$ , which correspond to  $4x^2 + 15x + 35$ . The labels "coeff.  $x^2$ ", "coeff.  $x$ ", and "const." are written below the quotient coefficients.

Quotient:  $4x^2 + 15x + 35$

Conclusion: Remainder  $r = 116$

Quotient:  $q(x) = 4x^2 + 15x + 35$

E.g. Divide

$$\underline{5x^4 + x^3 - 16x^2 + 1}$$

x term is missing

$$x + 2$$

Result: Remainder  $r = 9$ .

$$\text{Quotient} = q(x) = 5x^3 - 9x^2 + 2x - 4.$$

$$\underbrace{5x^4 + x^3 - 16x^2 + 1}_{\text{Divisor}} = \underbrace{(x+2)}_{\text{Dividend}} \cdot \underbrace{(5x^3 - 9x^2 + 2x - 4)}_{\text{quotient}} + \underbrace{9}_{\text{Remainder}}$$