Synthetic Division.

Monday, March 18, 2019 1:10 PM

Idea: $f(x) = x^3 + 5x^2 + 9x - 7$

$g(x) = x + 5.$

Want: $f(x) \div g(x)$ (or $\dfrac{f(x)}{g(x)}$)

Result of dividing $f(x)$ by $g(x)$ consists of the quotient $q(x)$ and the remainder $r(x)$

$$\underbrace{f(x)}_{\text{Divisor}} = \underbrace{g(x)}_{\text{Dividend}} \cdot \underbrace{q(x)}_{\text{quotient}} + \underbrace{r(x)}_{\text{remainder}}$$

or $$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

Q: How do we find the quotient $q(x)$ and the remainder $r(x)$ when we are given $f(x)$ and $g(x)$?

A: Synthetic Division.

Note: Synthetic Division only works if $g(x)$ has the form $g(x) = x - c$ or $g(x) = x + c$.

Process of Synthetic Division.

E.g. Find quotient and remainder for the division:

$$\frac{4x^3 + 3x^2 - 10x + 11}{x - 3}$$

Step 1:

$$\begin{array}{c|cccc} 3 & 4 & 3 & -10 & 11 \\ & & & & \\ \hline \end{array}$$

Step 2:

$$\begin{array}{c|cccc} 3 & 4 & 3 & -10 & 11 \\ & & 12 & 45 & 105 \\ \hline & 4 & 15 & 35 & \boxed{116} \end{array}$$

mult. mult. mult.

add add

coeff. x^2. coeff. x const.

Remainder

Quotient: $4x^2 + 15x + 35$

Conclusion: Remainder $\boxed{r = 116}$

Quotient: $\boxed{q(x) = 4x^2 + 15x + 35}$

Monday, March 18, 2019 1:26 PM

E.g. Divide

x term is missing

$$\frac{5x^4 + x^3 - 16x^2 + 1}{x+2}$$

$$\begin{array}{r|rrrrr}
-2 & 5 & 1 & -16 & 0 & 1 \\
 & & -10 & 18 & -4 & 8 \\
\hline
 & 5 & -9 & 2 & -4 & \boxed{9} \to R \\
 & x^3 & x^2 & x & \text{Const} &
\end{array}$$

mult.

mult.

mult.

add

Result: Remainder $r = 9$.

Quotient $= q(x) = 5x^3 - 9x^2 + 2x - 4$.

$$\underbrace{5x^4 + x^3 - 16x^2 + 1}_{\text{Divison}} = \underbrace{(x+2)}_{\text{Dividend}} \cdot \underbrace{(5x^3 - 9x^2 + 2x - 4)}_{\text{quotient}} + \underbrace{9}_{\text{Remainder}}$$