Test 3 Review

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MC.

(1) Distance between (1, -1) and (-3, 5)

Distance formula:

$$\boxed{D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Here,

$$D = \sqrt{(-3-1)^2 + (5-(-1))^2}$$

$$= \sqrt{16+36} = \sqrt{52} = \sqrt{4}\cdot\sqrt{13} = \boxed{2\sqrt{13}}$$

(2) Midpoint Formula:

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

Here:

$$\left(\frac{\frac{5}{4} + \frac{9}{4}}{2} ; \frac{-\frac{8}{5} + \frac{9}{5}}{2}\right)$$

$\rightarrow$ Simplify:

$$\left(\frac{\frac{14}{4}}{2} ; \frac{\frac{1}{5}}{2}\right) = \left(\frac{14}{8} ; \frac{1}{10}\right)$$

$$= \boxed{\left(\frac{7}{4} ; \frac{1}{10}\right)}$$

(3) $\boxed{\text{Standard Equation of circle: } (x-h)^2+(y-k)^2=R^2}$ Center (h,k) Radius R

Here, $h=3$, $k=5$

$$(x-3)^2+(y-5)^2=(\sqrt{19})^2$$

$$\rightarrow \boxed{(x-3)^2+(y-5)^2=19}$$

no x^3 term.

(4)

$$\begin{array}{c|cccccc} & 2 & -1 & \boxed{0} & 3 & -1 & 5 \\ 1 & & 2 & 1 & 1 & 4 & 3 \\ \hline & 2 & 1 & 1 & 4 & 3 & \boxed{8} \end{array}$$

Answer: Remainder $\boxed{R=8}$

Quotient $= \boxed{2x^4+x^3+x^2+4x+3}$.

(5)

$$\begin{array}{c|ccccc} 3 & 1 & 0 & 0 & 0 & -5 \\ & & 3 & 9 & 27 & 81 \\ \hline & 1 & 3 & 9 & 27 & 76 \end{array}$$

Answer: Remainder $=\boxed{76}$. Quotient $=\boxed{x^3+3x^2+9x+27}$.

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Typo: coeff. of x is 23

⑥ Step 1: Synthetic Division:

$$\begin{array}{c|cccc} \frac{2}{3} & 12 & -65 & 23 & 10 \\ & & 8 & -38 & -10 \\ \hline & 12 & -57 & -15 & \boxed{0} \end{array}$$

Step 2: Set quotient $=0$ → Solve for x.

$$12x^2 - 57x - 15 = 0$$

→ Divide both sides by 3:

$$4x^2 - 19x - 5 = 0$$

$$(4x+1)(x-5) = 0$$

→ $4x + 1 = 0$; $x - 5 = 0$

→ $x = -\frac{1}{4}$; $x = 5$

→ Solution set $= \boxed{\left\{ \frac{2}{3}, -\frac{1}{4}, 5 \right\}}$

⑦ Rational Zero Theorem.

$$\frac{p}{q} = \frac{p \text{ is a factor of constant term}}{q \text{ is a factor of leading coeff.}}$$

Here, constant term $= 2$

leading coeff. $= 6$

$$\frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$$= \left\{ \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{1}{6}, \pm \frac{2}{3} \right\}$$

8 Zeros: $2i, 4, -4$

By the Conjugate zero theorem: $-2i$ is also a zero

$$f(x) = (x - 2i)(x + 2i)(x - 4)(x + 4)$$

$$= (x^2 - (2i)^2)(x^2 - (4)^2)$$

Difference between squares

$i^2 = -1$

$$= (x^2 - 4i^2)(x^2 - 16)$$

$$= (x^2 + 4)(x^2 - 16)$$

$$= x^4 - 16x^2 + 4x^2 - 64$$

$$\boxed{f(x) = x^4 - 12x^2 - 64}$$

9 To find domain of rational function $f(x) = \dfrac{p(x)}{q(x)}$.

Set denom $q(x) = 0 \rightarrow$ Solve.

$$x^2 - 5x - 14 = 0$$

$$\rightarrow (x-7)\cdot(x+2) = 0$$

$$\rightarrow x = 7 \; ; \; x = -2.$$

Domain: in interval notation:

$$\boxed{(-\infty, -2) \cup (-2, 7) \cup (7, \infty)}$$

in set builder notation:

$$\boxed{\{x \mid x \neq 7 \text{ and } x \neq -2\}}$$

10 Vertical Asymptotes:

Step 1: Factor top and bottom completely.

$$\frac{x+6}{x^2-36} = \frac{x+6}{(x+6)(x-6)}$$

Step 2: Cancel (if there are common factor(s))

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$$\frac{\cancel{x+6}}{\cancel{(x+6)}(x-6)} = \frac{1}{x-6}$$

Step 3: Set denom (of simplified expression) $=0$

$$x-6=0 \rightarrow x=6$$

So, the V.A. $\boxed{x=6}$

11 Horizontal Asymptotes.

3 scenarios: 1 Deg top = Deg bottom.

$$\rightarrow y = \frac{\text{Leading coeff. of top}}{\text{leading coeff. of bottom}}$$

2 Deg top < Deg bottom.

$$\rightarrow y = 0$$

3 Deg top > Deg bottom

$\rightarrow$ No H.A.

Here, $g(x) = \dfrac{2x^2-4x-7}{7x^2-7x+5}$ $\begin{array}{l}\rightarrow \text{Deg top} = 2 \\ \rightarrow \text{Deg bottom} = 2\end{array}\Big\}$ equal

$\rightarrow$ Answer: H.A. $\boxed{y = \frac{2}{7}}$