Extra: Horizontal Stretch / Compression

A number is multiplied to the x in the function.

$$y = f(cx)$$

(c: constant, $c > 1$: compress; $0 < c < 1$: stretch)

$c > 1$ $\quad$ $0 < c < 1$

Compress or stretch the graph horizontally by a factor of c

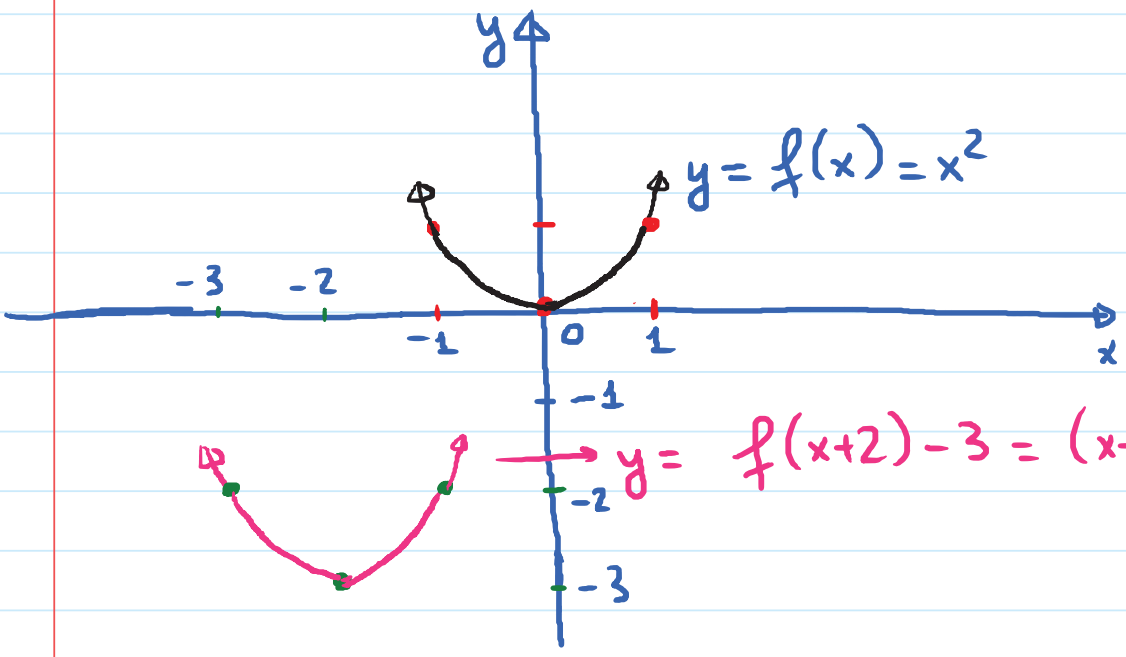
E.g. $y = x^2 + 7$.

* Basic graph: $y = x^2$
* Transformation: Shift basic graph 7 units up.

E.g. $y = (x+2)^2 - 3$.

* Basic graph: $y = x^2$
* Transformation: left 2 units, Down 3 units

Key points (original)		Key points (new)
$(0,0)$	→	$(-2,-3)$
$(1,1)$	→	$(-1,-2)$
$(-1,1)$	→	$(-3,-2)$



E.g. $y = 2 \cdot |x-1| + 3$.

* Basic function: $y = |x|$

* Transformations: Right 1, Stretch by factor of 2 vertically and up 3 units.

Key points (original) → Key points (new)

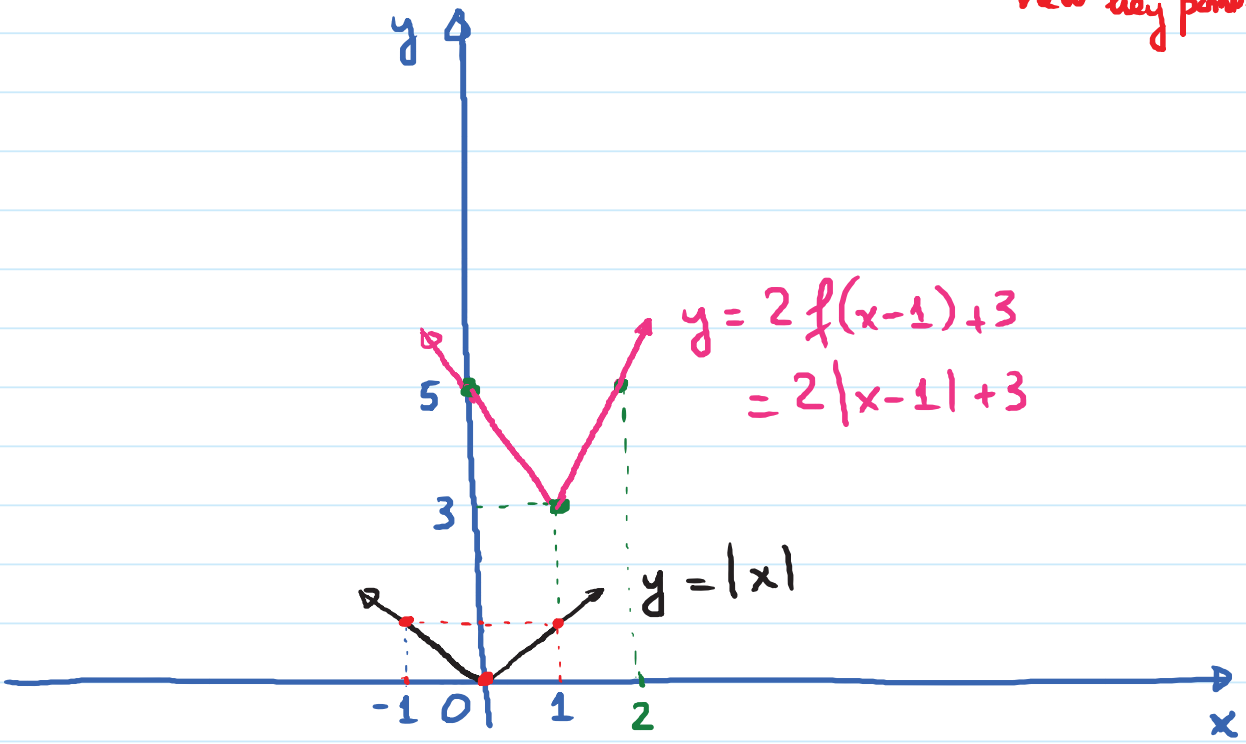
$(0,0) \xrightarrow{\text{right 1}} (1,0) \xrightarrow[\text{vertically}]{\text{Stretch}} (1,0) \xrightarrow[3]{\text{up}} (1,3)$

$(-1,1) \xrightarrow{\text{right 1}} (0,1) \xrightarrow[\text{vertically}]{\text{Stretch}} (0,2) \xrightarrow[3]{\text{up}} (0,5)$

$(1,1) \xrightarrow{\text{right 1}} (2,1) \xrightarrow[\text{Vertically}]{\text{Stretch}} (2,2) \xrightarrow[3]{\text{up}} (2,5)$

(factor 2)

new key points



E.g. $g(x) = -(x-5)^2 + 6$

Start with the graph $h(x) = x^2$.

Shift right 5 units, reflect across x-axis, shift up 6 units

E.g. Start with $f(x) = |x|$, shift left 2 units, up 9 units.

Formula of new function?

$$g(x) = |x+2| + 9$$

E.g. HW #26

$$g(x) = -\frac{1}{3} f(x-2) + 4.$$

Right 2, Compress vertically by $\frac{1}{3}$, Reflect across x-axis, up 4

Key points (original) Key points (new)

$(-6,0) \longrightarrow (-4,0) \longrightarrow (-4,0) \rightarrow (-4,0) \rightarrow \boxed{(-4,4)}$

$(-5,-6) \rightarrow (-3,-6) \rightarrow (-3,-2) \rightarrow (-3,2) \rightarrow \boxed{(-3,6)}$

$(-3,-6) \rightarrow (-1,-6) \rightarrow (-1,-2) \rightarrow (-1,2) \rightarrow \boxed{(-1,6)}$

$(0,0) \rightarrow (2,0) \rightarrow (2,0) \rightarrow (2,0) \rightarrow \boxed{(2,4)}$

$(2,9) \rightarrow (0,9) \rightarrow (0,3) \rightarrow (0,-3) \rightarrow \boxed{(0,1)}$

$(5,0) \rightarrow (3,0) \rightarrow (3,0) \rightarrow (3,0) \rightarrow \boxed{(3,4)}$