So, $\boxed{f(x) = 2(x+1)(x-2)(x-3)}$

$\to$ Expand : $2(x^2 - x - 2)(x-3)$

$= 2(x^3 - x^2 - 2x - 3x^2 + 3x + 6)$

$= 2(x^3 - 4x^2 + x + 6)$

$$\boxed{f(x) = 2x^3 - 8x^2 + 2x + 12}$$

Conjugate Zeros Theorem

If $a+bi$ is a zero of f, then $a-bi$ must be a zero of f.

In other words, nonreal zeros always appear in conjugate pairs.

E.g. Find all the zeros of

$$f(x) = x^3 - x^2 + 3x + 5.$$

Do the nonreal zeros appear in conjugate pairs?

1 List $= \{\pm 1, \pm 5\}$

2

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ & & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & \boxed{0} \end{array} \longrightarrow R = 0$$

$\rightarrow$ -1 is a zero

3

Set $q(x) = x^2 - 2x + 5 = 0$

Quadratic formula : $x = \dfrac{2 \pm \sqrt{4 - 20}}{2}$

$$x = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = \frac{\cancel{2}(1 \pm 2i)}{\cancel{2}}$$

$$x = 1 \pm 2i$$

Zeros: $\{-1, 1 + 2i, 1 - 2i\}$

Non real zeros appear in conjugate pair.

E.g. Find the polynomial of degree 3 that has 3 and $1+i$ as two of its zeros and has $\boxed{f(0)=5.}$

Sol: By the conjugate zero theorem $1-i$ is also a zero.

So, 3 zeros are: $3,\ 1+i,\ 1-i$

$$f(x) = a(x-3)(x-(1+i))(x-(1-i))$$

$$= a\cdot(x-3)\cdot(\underbrace{x-1}_{(A}\ \underbrace{-i}_{-\,B)})(\underbrace{x-1}_{(A}\ \underbrace{+i}_{+\,B)})$$

$$= a\cdot(x-3)\cdot\left[(x-1)^2 - i^2\right] \quad -1$$

$$= a\cdot(x-3)\left[(x-1)^2+1\right]$$

$$= a\cdot(x-3)(x^2-2x+1+1)$$

$$= a(x-3)(x^2-2x+2)$$

$$= a(x^3-2x^2+2x-3x^2+6x-6)$$

$$= a\cdot(\underbrace{x^3-5x^2+8x-6})$$

$$5 = f(0) = a\cdot(-6) \rightarrow a = -\frac{5}{6}.$$

$$f(x) = -\frac{5}{6}\left(x^3 - 5x^2 + 8x - 6\right)$$

of degree 4

E.g. Find the polynomial $f(x)$ with leading coeff. 1 and has i and $2i$ as 2 of its zeros

Sol: By the conjugate zero theorem, the zeros of f are:

$$i, -i, 2i, -2i$$

$$\text{So, } f(x) = \underset{\substack{\| \\ 1}}{\boxed{a}}\underbrace{(x-i)(x+i)}\underbrace{(x-2i)(x+2i)}$$

$$= (x^2 - i^2)(x^2 - 4i^2) \quad (i^2 = -1)$$

$$= (x^2 + 1)(x^2 + 4)$$

$$\boxed{f(x) = x^4 + 5x^2 + 4}$$