2.1. Trigonometric Functions of Acute Angle

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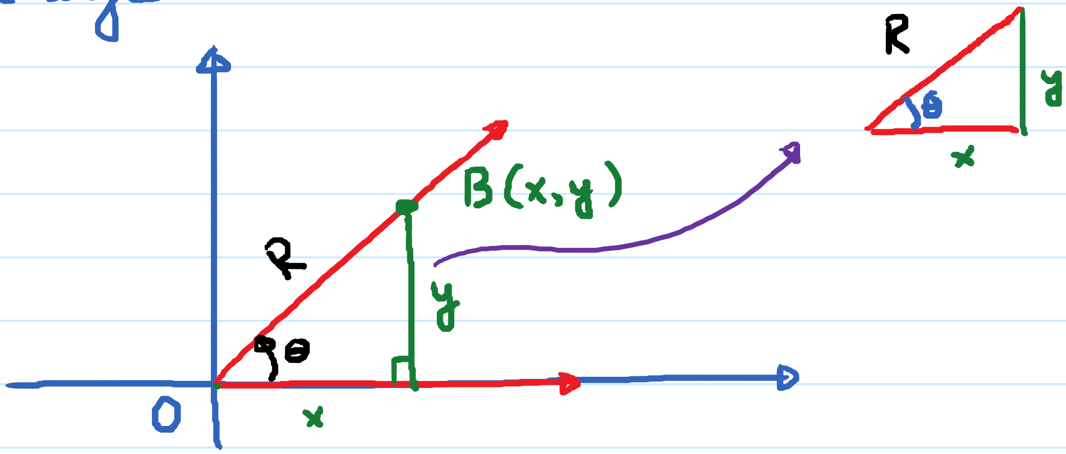
Acute angle → Q I



$$\sin\theta = \frac{y}{R} = \frac{\text{side opp. to } \theta}{\text{hypotenuse}} \; ; \; \cos\theta = \frac{x}{R} = \frac{\text{adj}}{\text{hyp.}}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{opp.}}{\text{adj.}} \; ; \; \cot\theta = \frac{x}{y} = \frac{\text{adj.}}{\text{opp.}}$$

$$\csc\theta = \frac{R}{y} = \frac{\text{hyp.}}{\text{opp.}} \; ; \; \sec\theta = \frac{R}{x} = \frac{\text{hyp.}}{\text{adj.}}$$

Right-triangle-based Definitions.

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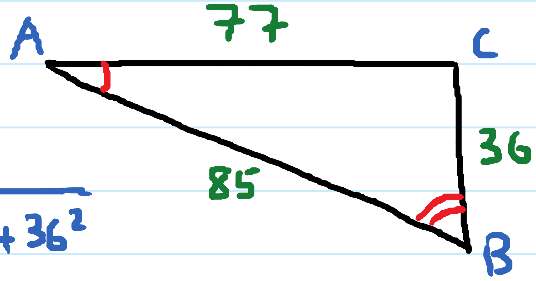
E.g.



$85 = \sqrt{77^2 + 36^2}$

Find $\sin A = \frac{36}{85}$

$\cos A = \frac{77}{85}$ $\quad \tan A = \frac{36}{77}$

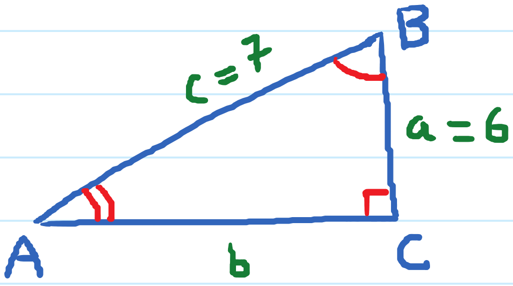
$\sin B = \frac{77}{85}$ $\quad \cos B = \frac{36}{85}$

$\tan B = \frac{77}{36}$

Ex. ABC is a right triangle with side lengths a, b, c

$m\angle C = 90^\circ$. Given: $a = 6$; $c = 7$.

Find $\csc B$, $\sec A$, $\tan A$, $\cot B$.



$$\csc B = \frac{hyp}{opp} = \frac{7}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{7\sqrt{13}}{13}}$$

$$\sec A = \frac{hyp}{adj} = \frac{7}{\sqrt{13}} = \frac{7\sqrt{13}}{13}$$

$$(6)^2 + b^2 = (7)^2$$

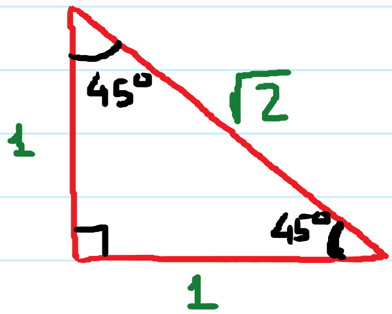
$$b = \sqrt{49-36} = \sqrt{13}$$

$$\tan A = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$$

$$\cot B = \frac{6\sqrt{13}}{13}$$

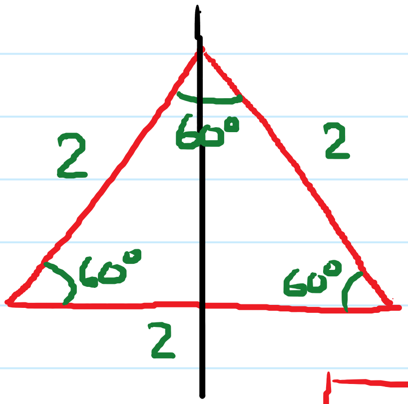
Trig Function Values of Special Triangles.

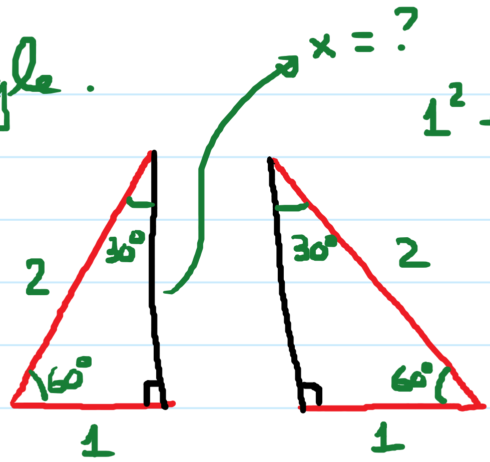
$45^\circ - 45^\circ - 90^\circ$ triangle



$$\boxed{\begin{array}{ll} \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} & \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \tan 45^\circ = 1 & \cot 45^\circ = 1 \\ \csc 45^\circ = \sqrt{2} & \sec 45^\circ = \sqrt{2} \end{array}}$$

$30° - 60° - 90°$ triangle.

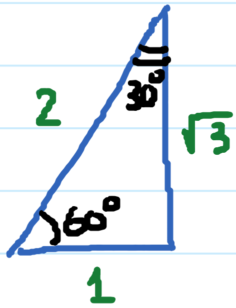




$1^2 + x^2 = 4$

$x^2 = 3$

$x = \sqrt{3}$



$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 30° = \frac{1}{2}$
$\cos 60° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\tan 60° = \sqrt{3}$	$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

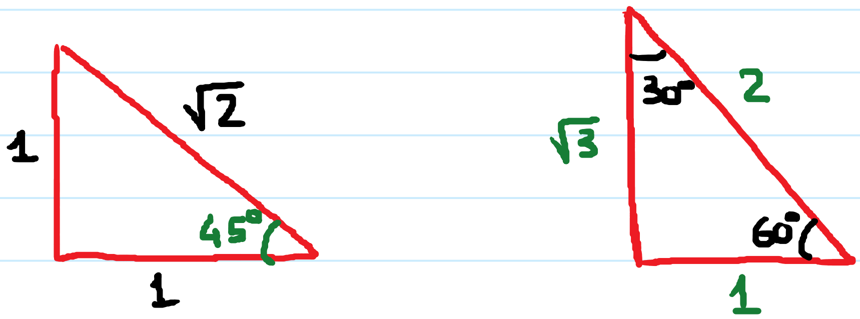
Ex. Find

$$\cos^2(420°) - \sin^2(-315°) + \cot(-330°)$$

$$(\cos 60°)^2 - (\sin 45°)^2 + \cot(30°)$$

$$\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 + \sqrt{3}$$

$$= \frac{1}{4} - \frac{2}{4} + \sqrt{3} = \boxed{-\frac{1}{4} + \sqrt{3}}$$



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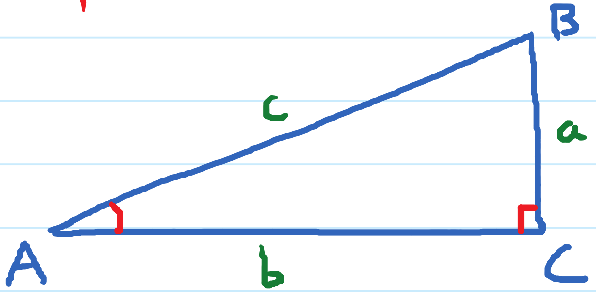
Cofunction Identities:



$\angle B = 90° - A$

$\angle A + \angle B = 90°$

A and B are complementary angles.

$\sin A = \frac{a}{c} = \cos B = \frac{a}{c}$

$\cos A = \sin B = \frac{b}{c}$

$\tan A = \cot B = \frac{a}{b}$

$\csc A = \sec B$

$\sec A = \csc B$

$\cot A = \tan B$.

Cofunction Identities:

For any acute angle A, the cofunction values of the complementary angles are equal.

$\sin A = \cos(90° - A)$; $\cos A = \sin(90° - A)$

$\sec A = \csc(90° - A)$; $\csc A = \sec(90° - A)$

$\tan A = \cot(90° - A)$; $\cot A = \tan(90° - A)$

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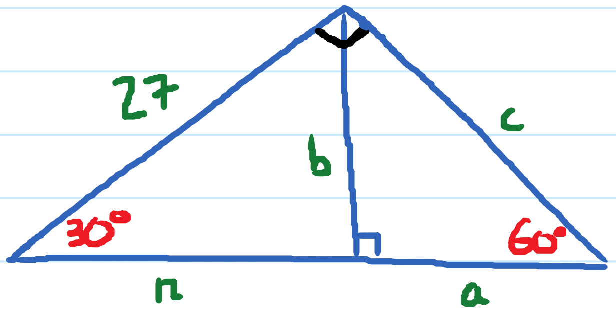
E.g. $\sin(9^\circ) = \cos(81^\circ)$

$\cot(76^\circ) = \tan(14^\circ)$

$\sec(65^\circ) = \csc(25^\circ)$

E.x.



$$\cos 30^\circ = \frac{n}{27} \longrightarrow \frac{\sqrt{3}}{2} = \frac{n}{27}$$

$$\longrightarrow n = \frac{27\sqrt{3}}{2}$$

$$\sin 30^\circ = \frac{b}{27} \longrightarrow \frac{1}{2} = \frac{b}{27}$$

$$\longrightarrow b = \frac{27}{2}$$

$$\tan 30^\circ = \frac{c}{27} \longrightarrow \frac{\sqrt{3}}{3} = \frac{c}{27}$$

$$\longrightarrow c = \frac{27\sqrt{3}}{3} = 9\sqrt{3}$$

$$\cos 60^\circ = \frac{a}{c} \longrightarrow \frac{1}{2} = \frac{a}{9\sqrt{3}} \longrightarrow a = \frac{9\sqrt{3}}{2}$$