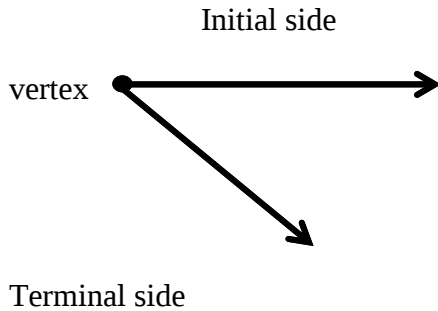
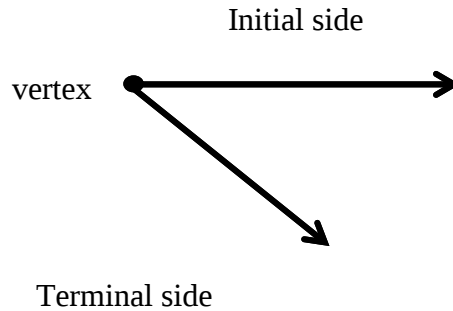


Angle - two rays with a common endpoint (vertex)

Positive Angle (counterclockwise rotation)

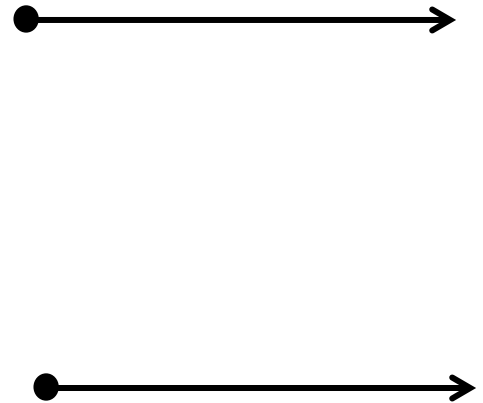
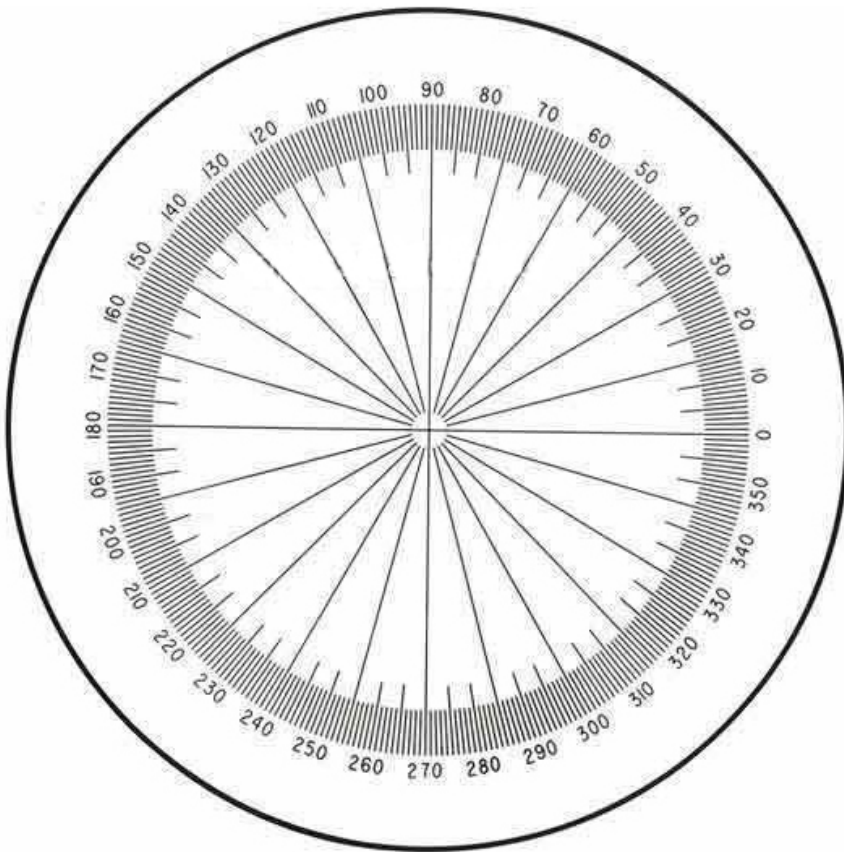


Negative Angle (clockwise rotation)



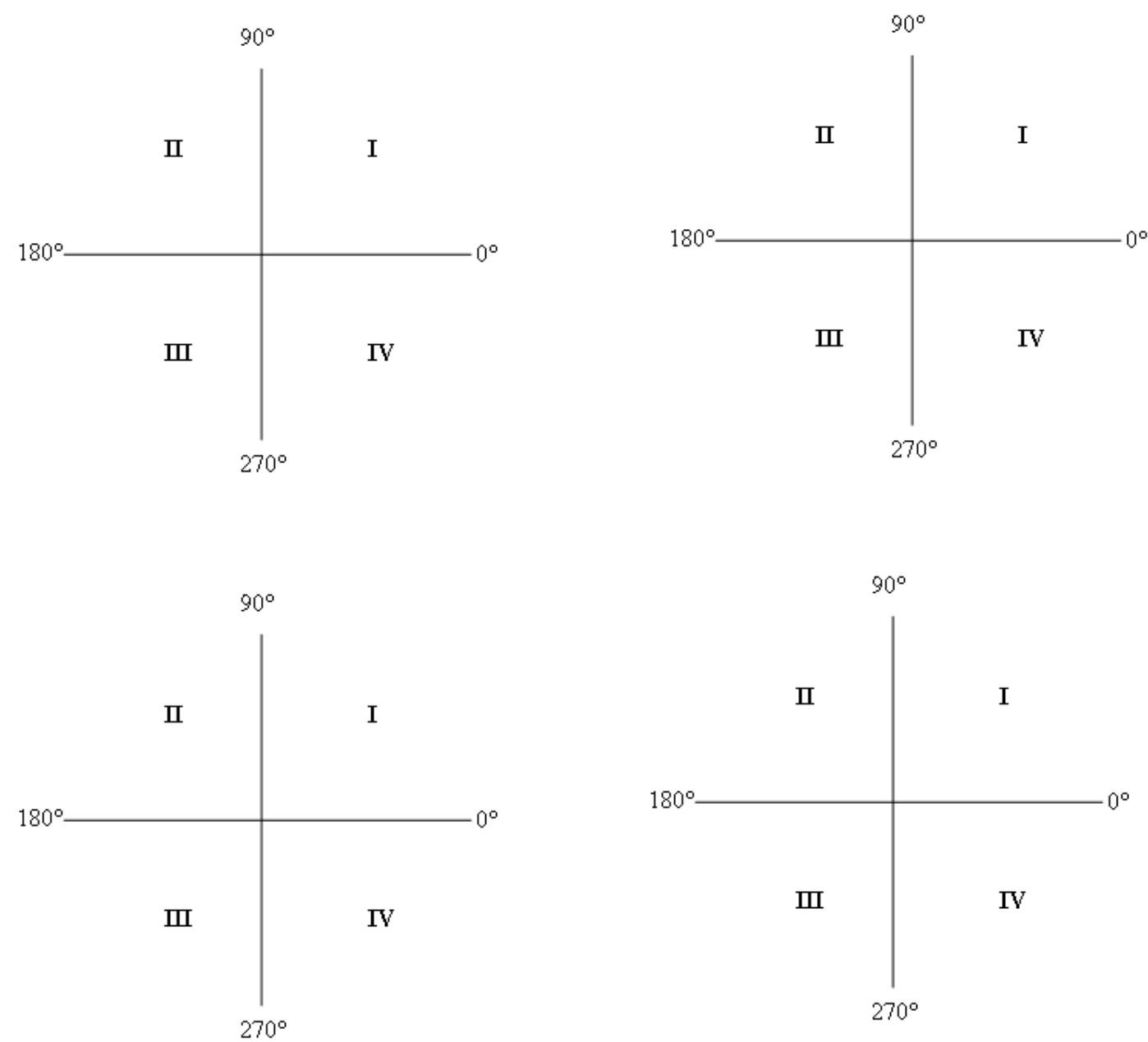
Degree Measure: The most common unit for measuring angles is the **degree**.

A complete rotation of a ray gives an angle of 360 degrees or 360°



Standard position

Definition: An angle is in **standard position** if its vertex is at the origin and its initial side lies on the positive x-axis. The angle is said to lie in the quadrant in which its terminal side lies.



Definitions:

Type of angle: Refers to an angle measuring:

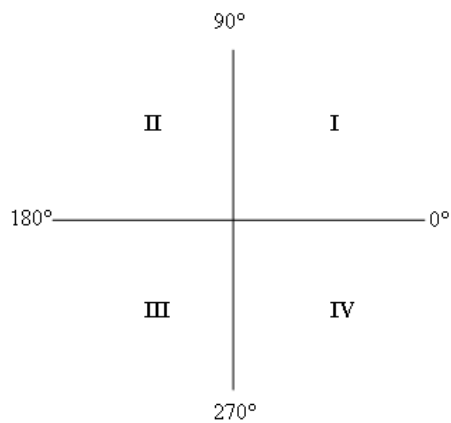
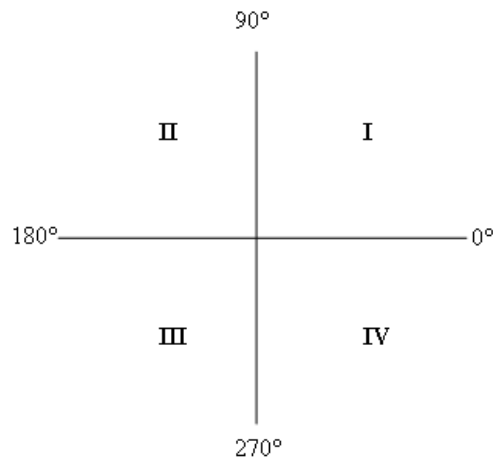
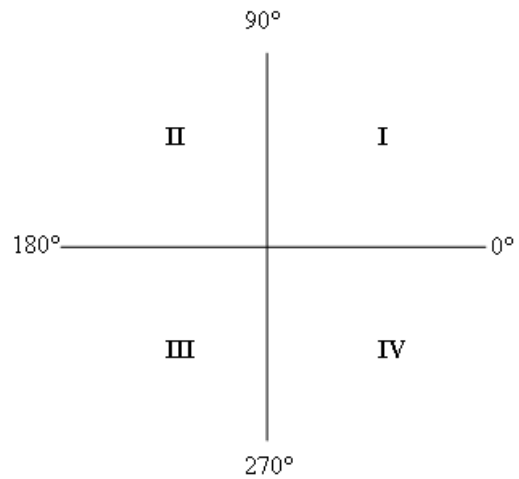
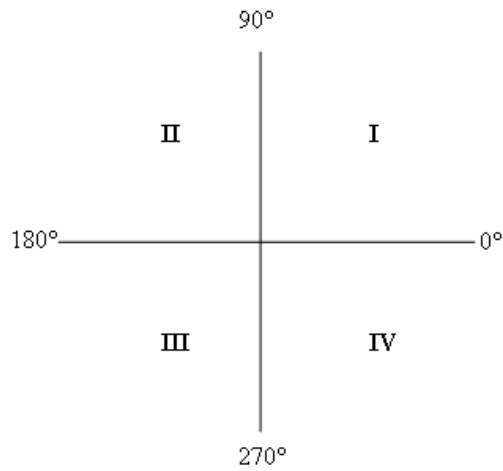
Acute	Between 0° and 90°
Right	Exactly 90°
Obtuse	Between 90° and 180°
Straight	Exactly 180°

Definition: **Complementary** angles: Two positive angle measures that add up to be 90° .

Definition: **Supplementary** angles: Two positive angle measures that add up to be 180° .

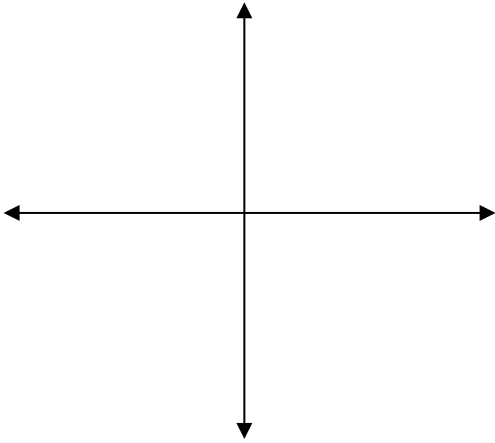
NOTE: We often use the Greek letters such as θ (theta), α (alpha), and β (beta) to name angles.

Defintion: **Quadrantal** angles are angles whose terminal side lies on the x-axis or y-axis.

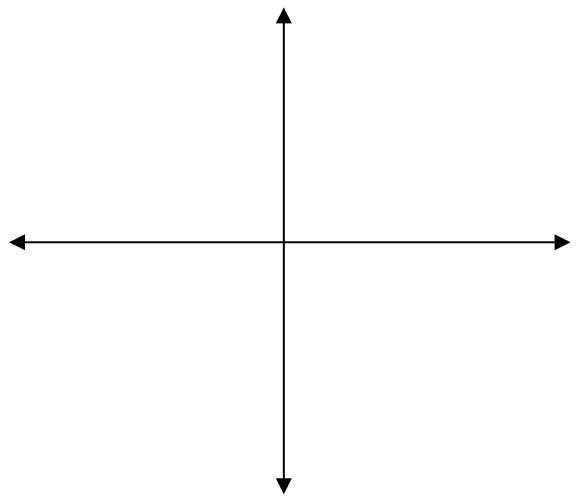
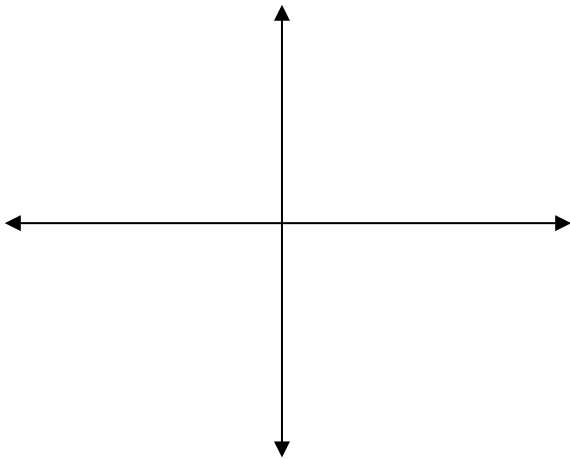


Definition: **Coterminal** angles are angles that have the same initial side and the same terminal side, but different amounts of rotation. Their measures differ by a multiple of 360° .

Sketch $\theta = 60^\circ$ in standard position

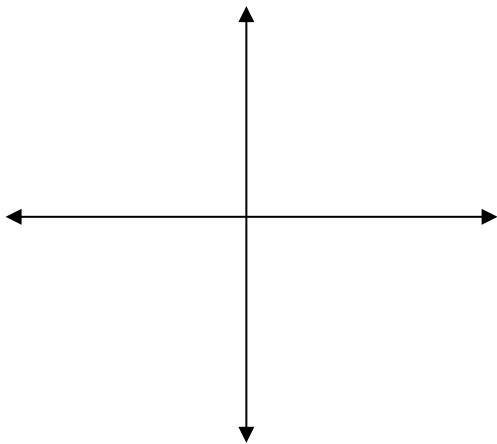


a) Find the measure a positive coterminal angle. b) Find the measure of a negative coterminal angle.

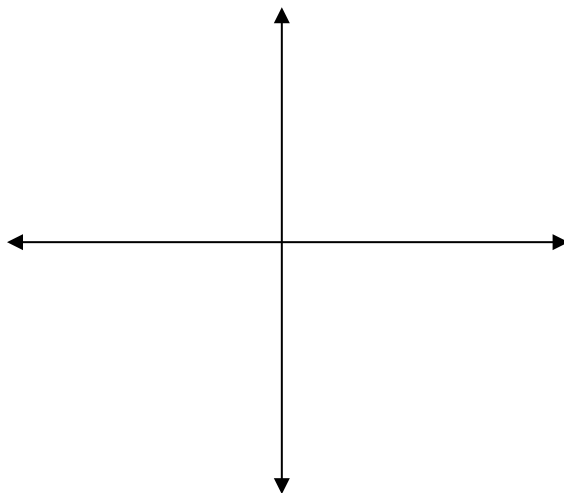
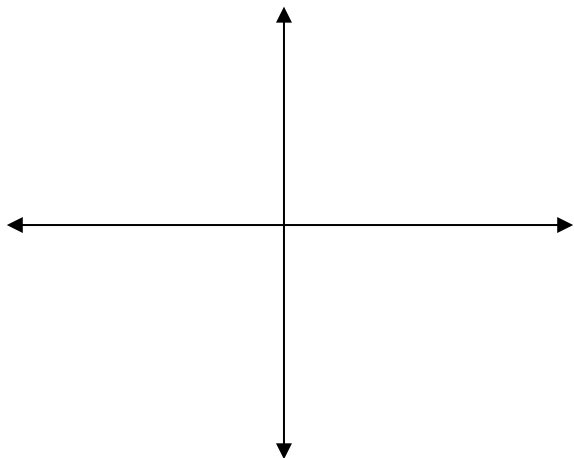


We can obtain any angle coterminal with 60° by adding or subtracting an integer multiple of 360° .

Sketch $\theta = -200^\circ$ in standard position.



- a) Find the measure a positive coterminal angle. b) Find the measure of a negative coterminal angle.



We can obtain any angle coterminal with -200° by adding or subtracting an integer multiple of 360° .

Finding Coterminal Angles

We can find coterminal angles by adding or subtracting a complete rotation (360°) to/from the given angle.

We can use the expression, $\theta + n \cdot 360^\circ$, where n is any integer, $n = \dots -3, -2, -1, 0, 1, 2, 3 \dots$

This is equivalent to adding or subtracting 360° n times to/from the given angle θ .

Example: Use the expression, $\theta + n \cdot 360^\circ$, to give two positive and two negative angles that are coterminal to 135° .

Example: Find the angle of least positive measure (not equal to the given angle) that is coterminal with each angle.

a) 1106°

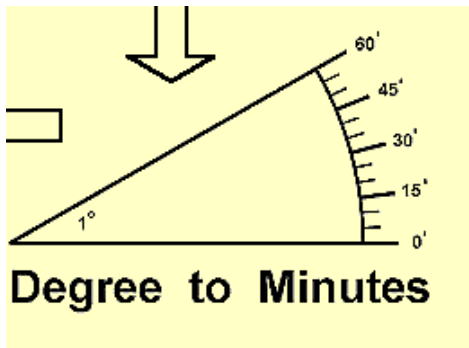
b) -150°

c) -603°

d) 86°

Degrees, Minutes, and Seconds

1 degree is made up of 60 smaller parts called **minutes**.

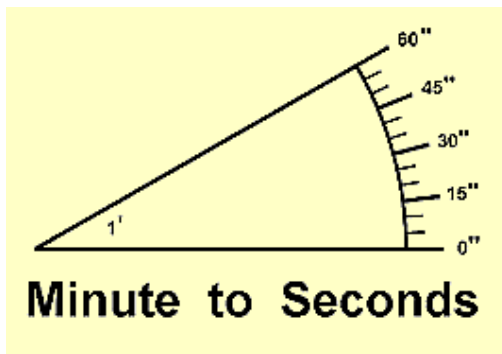


$$1 \text{ degree} = 60 \text{ minutes} \rightarrow 1^\circ = 60'$$

AND

$$1 \text{ minute} = \frac{1}{60} \text{ degree} \rightarrow 1' = \frac{1}{60}^\circ$$

1 minute is made up of 60 smaller parts called **seconds**.



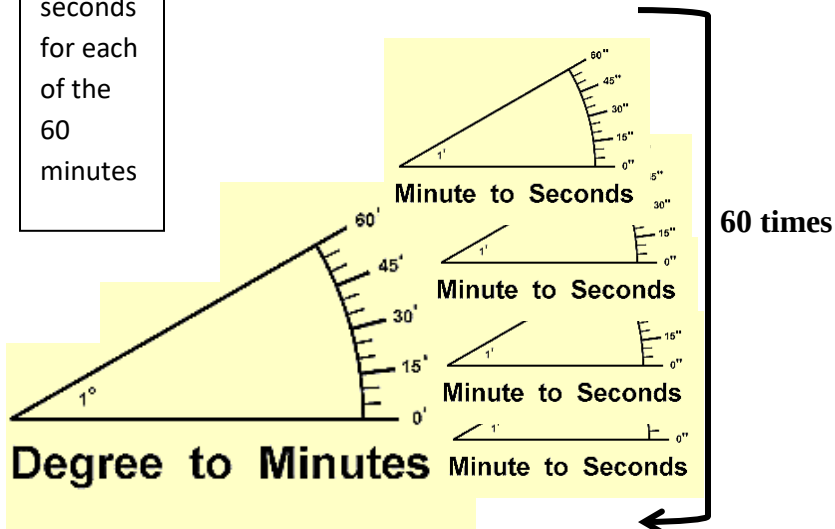
$$1 \text{ minute} = 60 \text{ seconds} \rightarrow 1' = 60''$$

AND

$$1 \text{ second} = \frac{1}{60} \text{ minute} \rightarrow 1'' = \frac{1}{60}'$$

Question: How many seconds make up 1 degree?

60
seconds
for each
of the
60
minutes



$$1 \text{ degree} = \underline{\hspace{2cm}} \text{ seconds}$$

AND

$$1 \text{ second} = \underline{\hspace{2cm}} \text{ degree}$$

Calculations with Degrees, Minutes, and Seconds

Example: Perform each calculation.

a) $28^{\circ} 35' + 63^{\circ} 52'$

b) $180^{\circ} - 117^{\circ} 29'$

c) $73^{\circ} 23' - 47^{\circ} 48'$

Converting an angle measure to decimal degrees (to the nearest thousandth)

$105^{\circ} 20' 32''$

To convert to decimal degrees:

- 1) The degrees should be placed as written before the decimal.
- 2) Divide the minutes by 60 and the seconds by 3600.
- 3) Input into calculator. (don't round yet)
- 4) Add together and round to the nearest thousandth.

Converting decimal degrees into degrees, minutes, and seconds

85.263°

To convert to degrees, minutes, and seconds:

- 1) The part before the decimal is the degrees.
- 2) Multiply the decimal portion by 60'. This portion is now converted into minutes. The part before the decimal is now the whole minutes.
- 3) Multiply the remaining decimal portion by 60''. This portion now converted into seconds. Round to the nearest whole second.