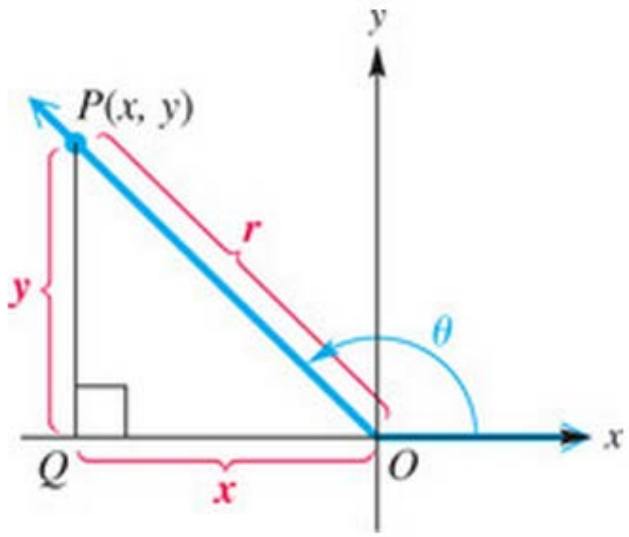
Section 1.3 Trigonometric Functions



θ is any angle in standard position

P is any point (x, y) on the terminal side of the angle

r is the distance from the origin to the point P

Using the distance formula, we know that

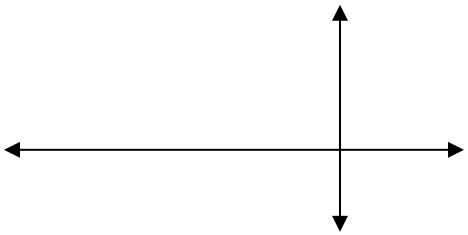
$r = \sqrt{x^2 + y^2}$ *and* $r > 0$

The six trigonometric functions are defined as follows

(sine)	(cosine)	(tangent)
$\boldsymbol{\sin\theta = \frac{y}{r}}$	$\boldsymbol{\cos\theta = \frac{x}{r}}$	$\boldsymbol{\tan\theta = \frac{y}{x}}\ (x \neq 0)$
$\boldsymbol{\csc\theta = \frac{r}{y}}\ (y \neq 0)$	$\boldsymbol{\sec\theta = \frac{r}{x}}\ (x \neq 0)$	$\boldsymbol{\cot\theta = \frac{x}{y}}\ (y \neq 0)$
(cosecant)	(secant)	(cotangent)

Example 1:

The terminal side of an angle θ in standard position passes through the point $(-12, 5)$. Find the values of the six trigonometric functions.



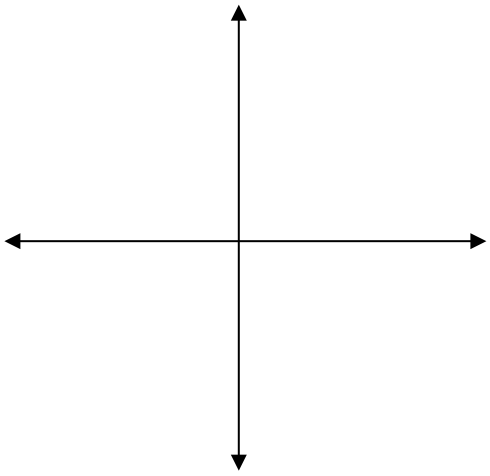
$\sin\theta =$ $\cos\theta =$

$\csc\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

Example 2:

The terminal side of an angle θ in standard position passes through the point $(-2\sqrt{3}, -2)$. Find the values of the six trigonometric functions.



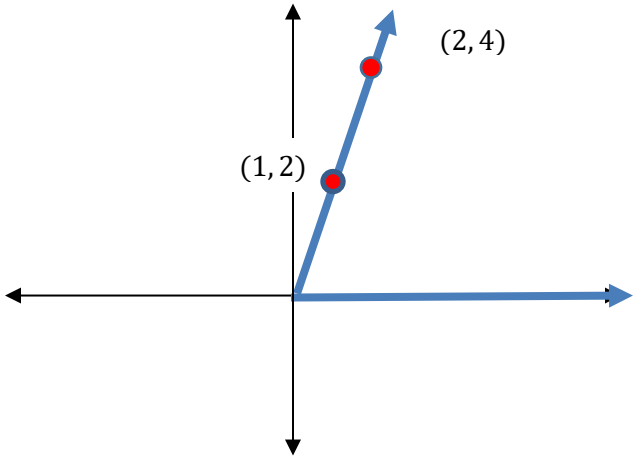
$\sin\theta =$ $\cos\theta =$

$\csc\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

NOTE: We can pick **ANY** point on the terminal side to find the values of the six functions.

For the following angle θ , show that the points $(1, 2)$ and $(2, 4)$ will give the same values for the six functions.



The terminal side of an angle θ in standard position passes through the point $(1, 2)$. Find the function values.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

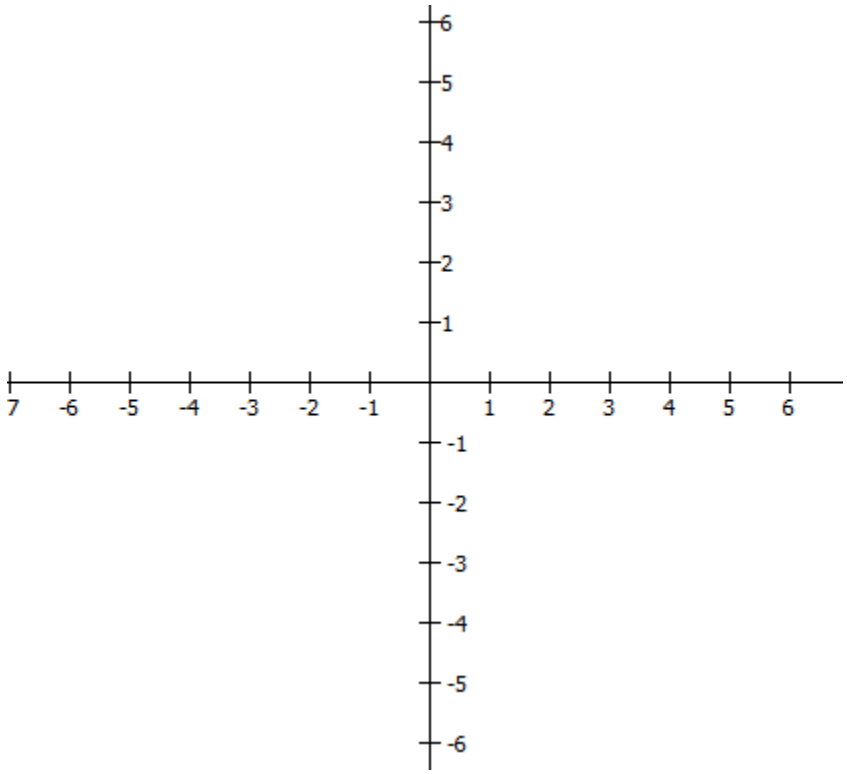
The terminal side of an angle θ in standard position passes through the point $(2, 4)$. Find the function values.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

Example 3:

An equation of the terminal side of an angle θ in standard position is given with a restriction on x. Sketch the least positive such angle θ, and find the value of the six trigonometric functions of θ.

$-5x - 3y = 0, \qquad x \leq 0$

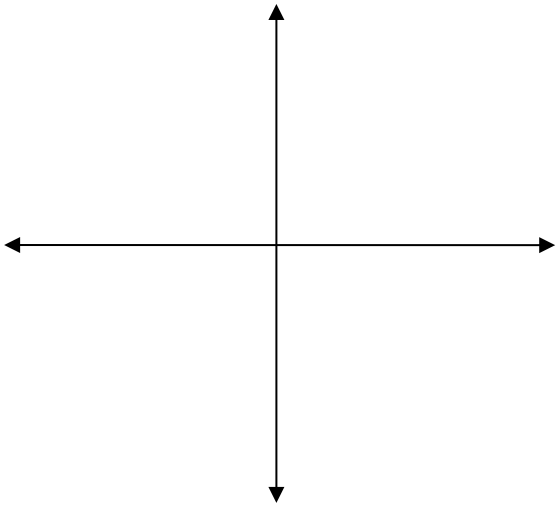


$\sin\theta =$ $\cos\theta =$

$\csc\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

The trigonometric function values of the quadrantal angles.



θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°						
90°						
180°						
270°						

NOTE: Coterminal angles have the same trigonometric function values.

Example 4:

Find the indicated function values for the following.

a) $\sin(-270°)$ b) $\tan 1800°$ c) $\cot 540°$

Example 5: Evaluate the following expressions.

a) $\tan 0° - 6\sin 90°$ b) $\cos^2(-180°) + \sin^2(-180°)$