Section 1.4 Using the Definitions of the Trigonometric Functions

We saw in the previous section that some of the trigonometric functions are reciprocals of each other.

Multiplying reciprocals together always results in a value of 1.

$(sin\,\theta)(csc\,\theta)$ =

This leads us to what we call the **reciprocal identities.**

$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Example 1: Find each function value.

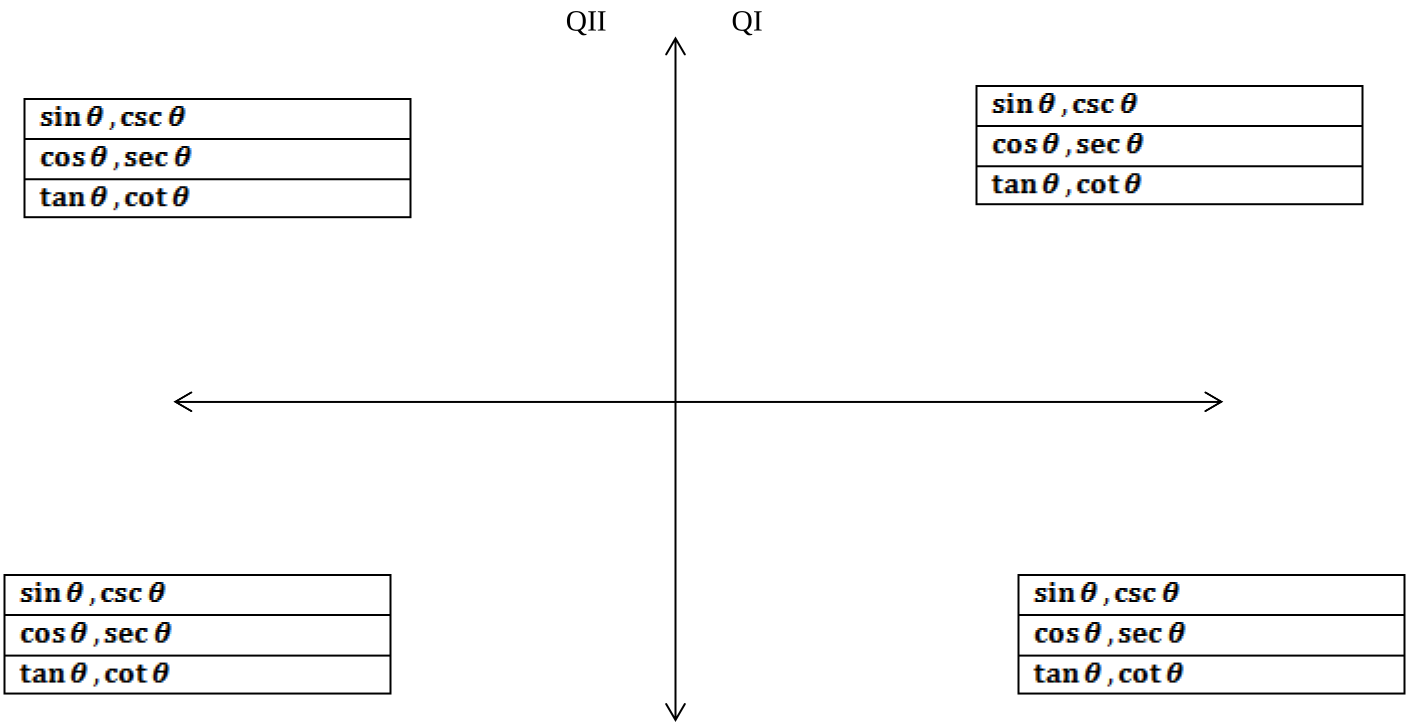
a) Find $\tan\theta$, given that $\cot\theta = 4$.

b) Find $\sec\theta$, given that $\cos\theta = -\frac{2}{\sqrt{20}}$

c) Find $\cos\theta$, given that $\sec\theta = 9.80425133$

NOTE: Reciprocals always have the same sign.

<u>**Determining the signs of the trigonometric functions of nonquadrantal angles**</u>



Example 2: Identify the quadrant(s) of an angle θ that satisfies the given conditions.

a) $\cos\theta < 0, \quad \sin\theta < 0$

b) $\cos\theta > 0, \quad \sec\theta > 0$

c) $\cot\theta < 0, \quad \sec\theta < 0$

<u>**Example 3:**</u> Find the signs of the six trigonometric functions for the given angle.

a) $-115°$

b) $855°$

sin $(-115°)$ ____

cos $(-115°)$ ____

tan $(-115°)$ ____

cot $(-115°)$ ____

sec $(-115°)$ ____

csc $(-115°)$ ____

sin $855°$ ____

cos $855°$ ____

tan $855°$ ____

cot $855°$ ____

sec $855°$ ____

csc $855°$ ____

<u>Pythagorean Identities</u>

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

We know form the Pythagorean Theorem that $y^2 + x^2 = r^2$.

NOTE: It is important to be able to transform these identities into their equivalent forms.

<u>Quotient Identities</u>

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Using Identities to find Function Values

Example 4: Find $\sin\theta$, given that $\cos\theta = \frac{4}{5}$ and θ is in quadrant IV.

Example 5: Find $\tan\theta$, given that $\sin\theta = \frac{1}{2}$ and θ is in quadrant II.

Example 6: Find the five remaining trigonometric function values for each angle θ.

a) $\sin\theta = \frac{\sqrt{2}}{6}$, and $\cos\theta < 0$

$\cos\theta =$ ____

$\tan\theta =$____

$\cot\theta =$ ____

$\sec\theta =$ ____

$\csc\theta =$ ____

b) $\cos\theta = -\frac{\sqrt{3}}{2}$, and θ is in quadrant III

$\sin\theta =$ ____

$\tan\theta =$____

$\cot\theta =$ ____

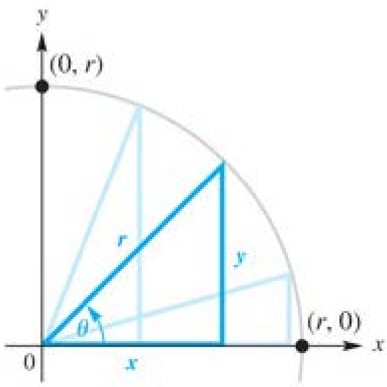
$\sec\theta =$ ____

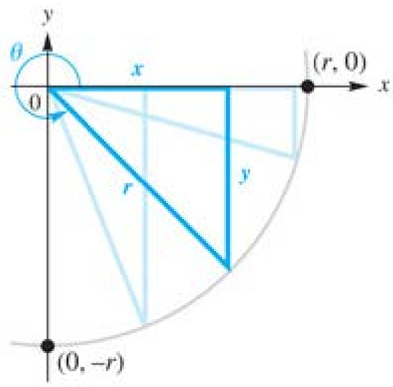
$\csc\theta =$ ____

The Range Values of the six trigonometric functions (Output values of the functions)

Trigonometric Function of $\boldsymbol{\theta}$	Range
$\sin\boldsymbol{\theta}, \cos\boldsymbol{\theta}$	$[-\mathbf{1}, \mathbf{1}]$
$\csc\boldsymbol{\theta}, \sec\boldsymbol{\theta}$	$(-\infty, -\mathbf{1}] \cup [\mathbf{1}, \infty)$
$\tan\boldsymbol{\theta}, \cot\boldsymbol{\theta}$	$(-\infty, \infty)$

Since r always represents the longest side of the triangle,





The sides x and y can vary greatly in relationship to each other.

$y = x$ OR $y < x$ OR $y > x$

Example 7: Decide whether each statement is possible or impossible for some angle θ.

a) $\sin\theta = 3$

b) $\cos\theta = -0.96$

c) $\csc\theta = 100$

d) $\cos\theta = -2$ *and* $\sec\theta = -\frac{1}{2}$

e) $\csc\theta = -2$ *and* $\sin\theta = -\frac{1}{2}$