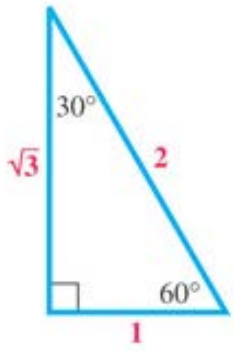
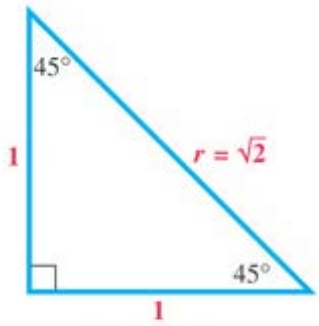
Section 2.2 Trigonometric Functions of Non-Acute Angles

Make sure that you know the special triangles!

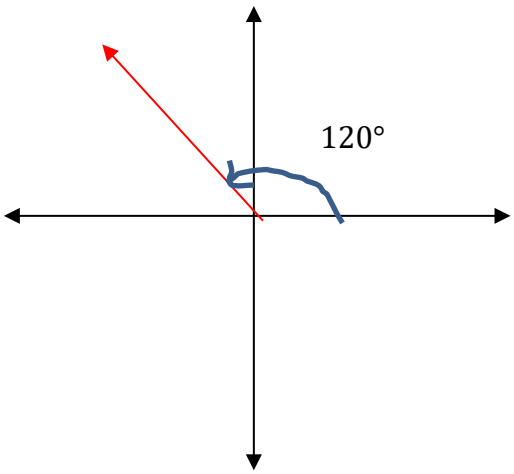


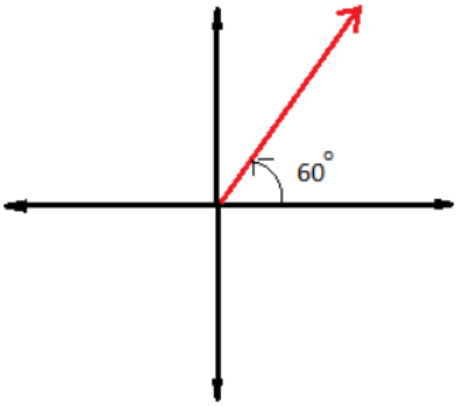


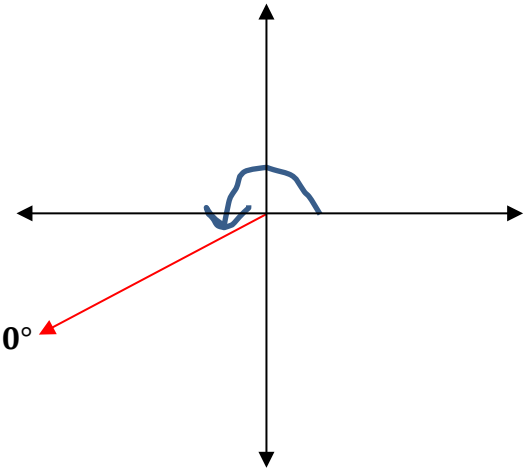
45°–45° right triangle

Reference Angles

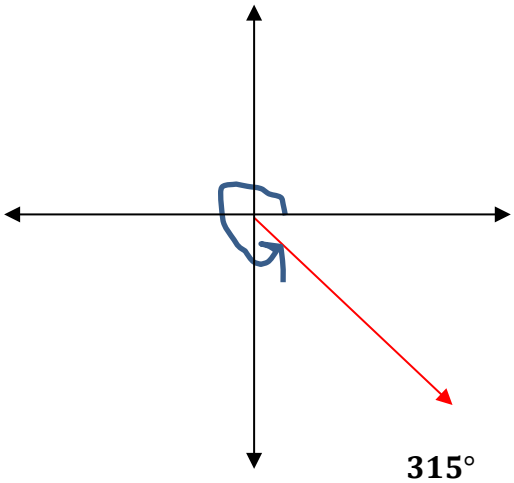
A **reference angle** is the positive acute angle made by the terminal side and the **x-axis.**





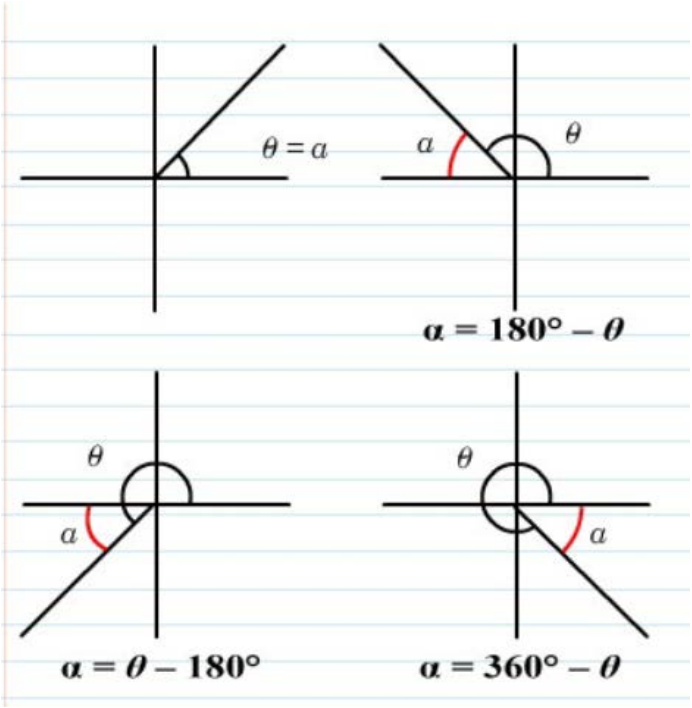


210°



315°

For any angle θ (between 0° *and* 360°), we can find the reference angle α by using the following table.



Example 1: Find the reference angle for the following angles.

a) 294°

b) 142°

c) 207°

d) 1130°

Finding Trigonometric Function Values for Any Nonquadrantal Angle θ

Step 1 If $\theta > 360°$, or if $\theta < 0°$, then find a coterminal angle by adding or subtracting $360°$ as many times as needed to get an angle greater than $0°$ but less than $360°$.

Step 2 Find the reference angle θ'.

Step 3 Find the trigonometric function values for reference angle θ'.

Step 4 Determine the correct signs for the values found in Step 3. (Use the table of signs in **Section 1.4,** if necessary.) This gives the values of the trigonometric functions for angle θ.

We can use the reference angle to find values of the six trigonometric functions for non-acute angles.

Example 2: Find the values of the six trigonometric functions for 240°.

$\sin 240° =$ $\csc 240° =$

$\cos 240° =$ $\sec 240° =$

$\tan 240° =$ $\cot 240° =$

Example 3: Find the values of the six trigonometric functions for 150°.

$\sin 150° =$ $\csc 150° =$

$\cos 150° =$ $\sec 150° =$

$\tan 150° =$ $\cot 150° =$

Example 4: Find the values of the six trigonometric functions for 315°.

$\sin 315° =$

$\cos 315° =$

$\tan 315° =$

$\csc 315° =$

$\sec 315° =$

$\cot 315° =$

Example 5: Find the values of the six trigonometric functions for 420°.

$\sin 420° =$

$\cos 420° =$

$\tan 420° =$

$\csc 420° =$

$\sec 420° =$

$\cot 420° =$

Example 6: Find the exact value of each expression.

a) $\sin(-150°)$

b) $\cot 1035°$

c) $\cos(-300°)$

d) $\sec(750°)$

Example 7: Find all values of θ, if θ is in the interval $[0°, 360°)$ and has the given function value.

a) $\cot\theta = -\sqrt{3}$

b) $\cos\theta = -\frac{\sqrt{2}}{2}$

c) $\csc\theta = \frac{2\sqrt{3}}{3}$

Know the Reference angle families from $0°$ to $360°$. The angles in each reference angle family will have the same function values, except for the signs. The sign of the values are based upon the quadrant of the angle.

Reference Angle	Reference angle family of angles	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
30°	30°, 150°, 210°, 330°	$\pm\frac{1}{2}$	$\pm\frac{\sqrt{3}}{2}$	$\pm\frac{\sqrt{3}}{3}$	$\pm\sqrt{3}$	$\pm\frac{2\sqrt{3}}{3}$	±2
45°	45°, 135°, 225°, 315°	$\pm\frac{\sqrt{2}}{2}$	$\pm\frac{\sqrt{2}}{2}$	±1	±1	$\pm\sqrt{2}$	$\pm\sqrt{2}$
60°	60°, 120°, 240°, 300°	$\pm\frac{\sqrt{3}}{2}$	$\pm\frac{1}{2}$	$\pm\sqrt{3}$	$\pm\frac{\sqrt{3}}{3}$	±2	$\pm\frac{2\sqrt{3}}{3}$