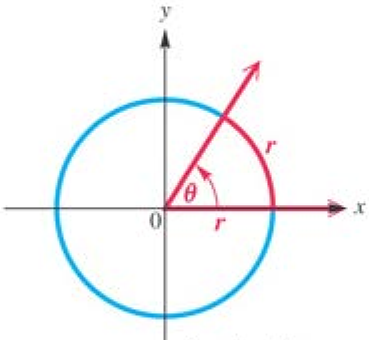
Section 3.1 # Radian Measure

Radian measure enables us to treat the trigonometric functions as functions with domains (inputs) of REAL NUMBERS, rather than angles.

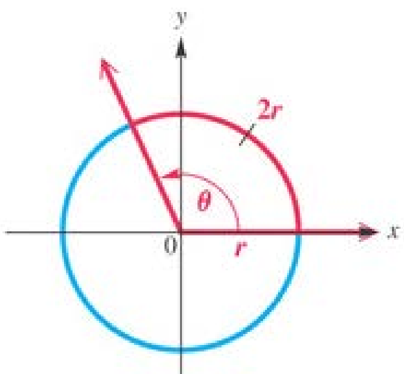


$\theta = 1$ radian

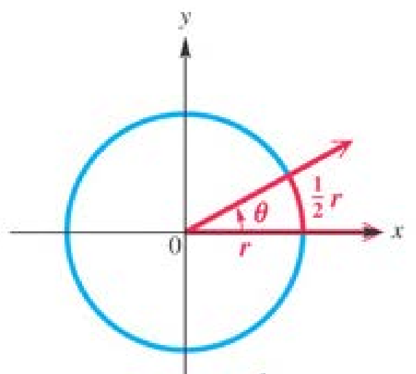
If $\boldsymbol{\theta}$ is a central angle of a circle of radius $\boldsymbol{r}$**,** and θ intercepts an arc of length $\boldsymbol{s}$**,** then the radian measure of θ is found by,

$$\boldsymbol{\theta = \frac{s}{r}}$$

An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of **1 radian.**

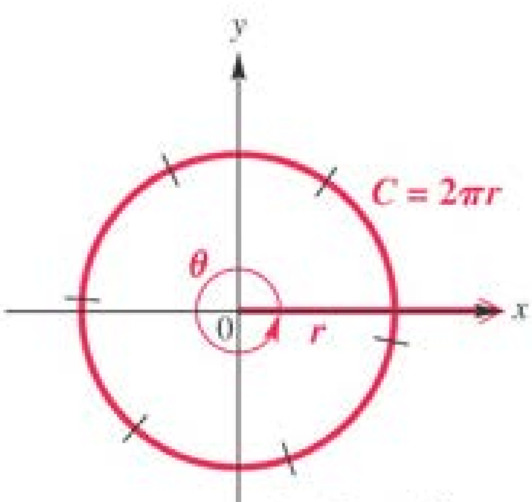


$\theta = 2$ radians



$\theta = \frac{1}{2}$ radian

How many radians represents a complete rotation?



$\theta = 2\pi$ radians

Converting between degrees and radians

Since **180° = $\boldsymbol{\pi}$ radians** ,

$1 \text{ degree} = \frac{\pi}{180}$ radians and $1 \text{ radian} = \frac{180}{\pi}$ degrees

To convert from

1) degrees to radians, multiply by $\frac{\pi}{180}$

2) radians to degrees, multiply by $\frac{180}{\pi}$

Example 1: Convert each degree measure to radians. Show as a multiple of π and give its decimal approximation.

a) $60°$

b) $-135°$

c) $325.7°$

Example 2: Convert each radian measure to degrees.

a) 2.92

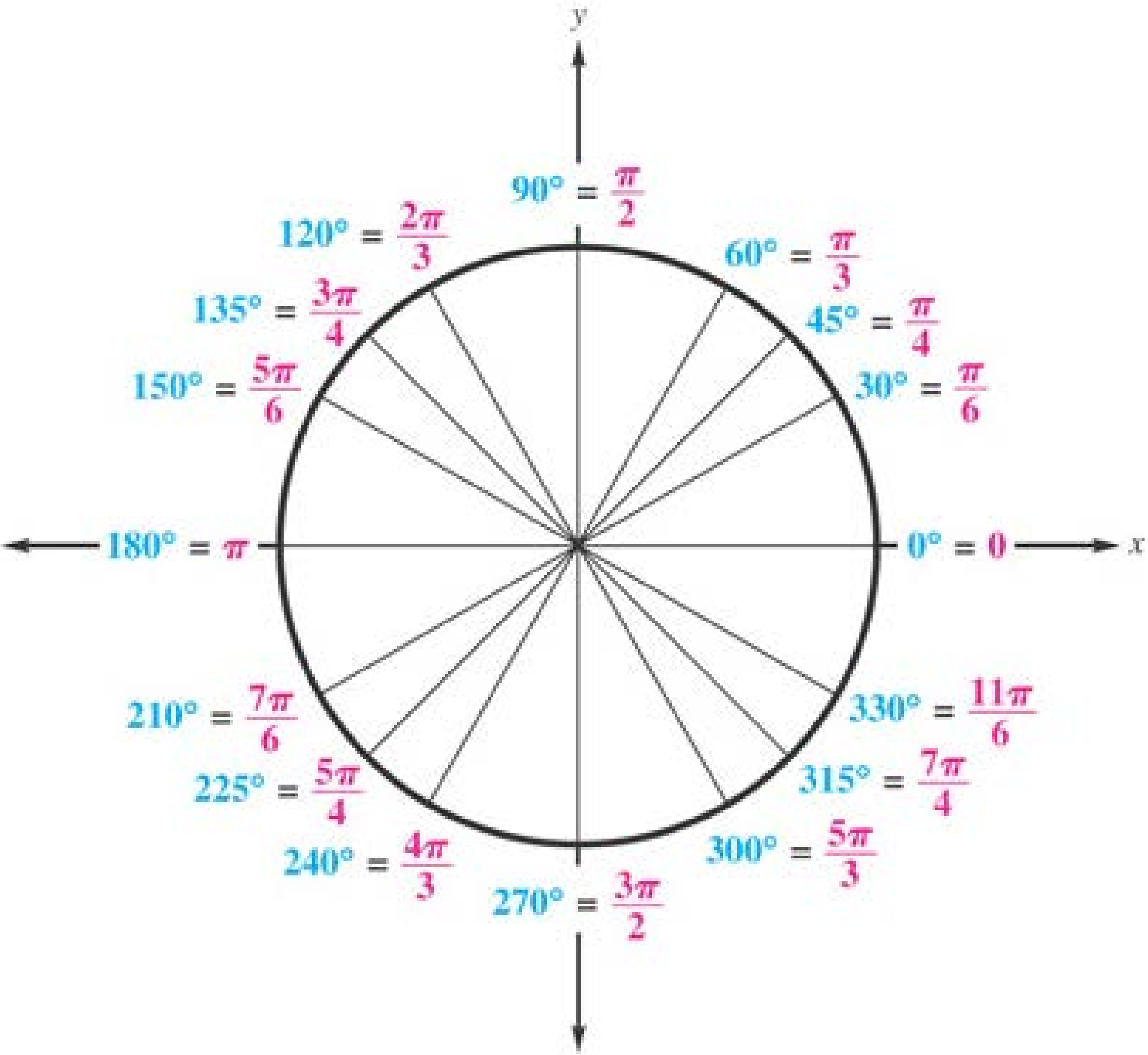
NOTE: If no unit of measurement is specified for an angle, then the angle is understood to be in radians.

NOTE: A shortcut to convert radians to degrees: substitute 180° for π.

b) $\frac{\pi}{6}$

c) $\frac{13\pi}{3}$

Important degree measures and their radian equivalences



Reference angles are super easy to recognize for the radian measures shown here.

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ Reference angle: $\frac{\pi}{6}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ Reference angle: $\frac{\pi}{4}$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ Reference angle: $\frac{\pi}{3}$

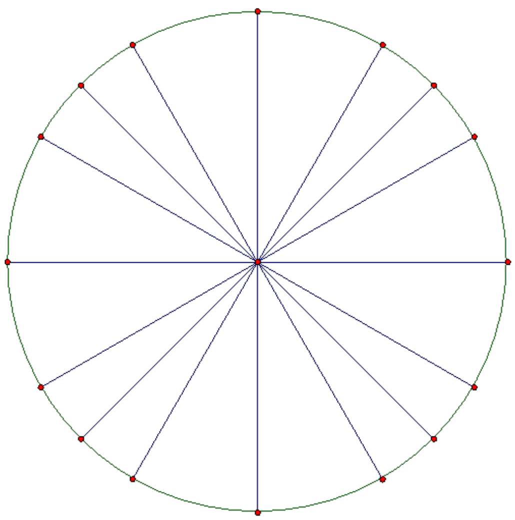
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$30° = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$45° = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60° = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$

Example 3: Find the exact value of each expression without using a calculator.

a) $\tan\frac{2\pi}{3}$ = ____________

Quadrant:

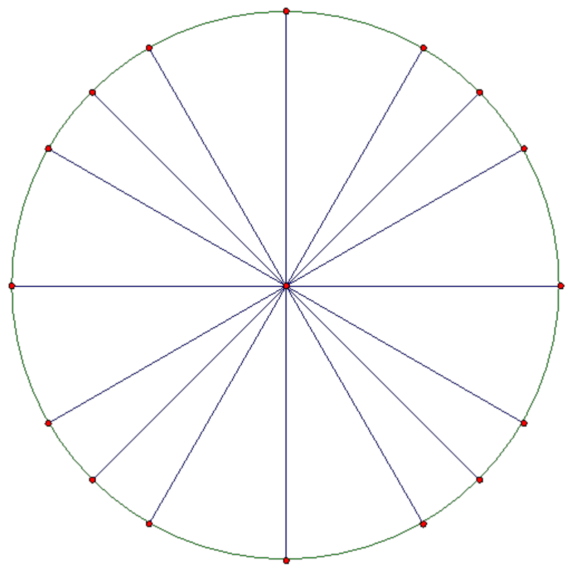
Reference Angle:



b) $\sec\frac{7\pi}{4}$ = ___________

Quadrant:

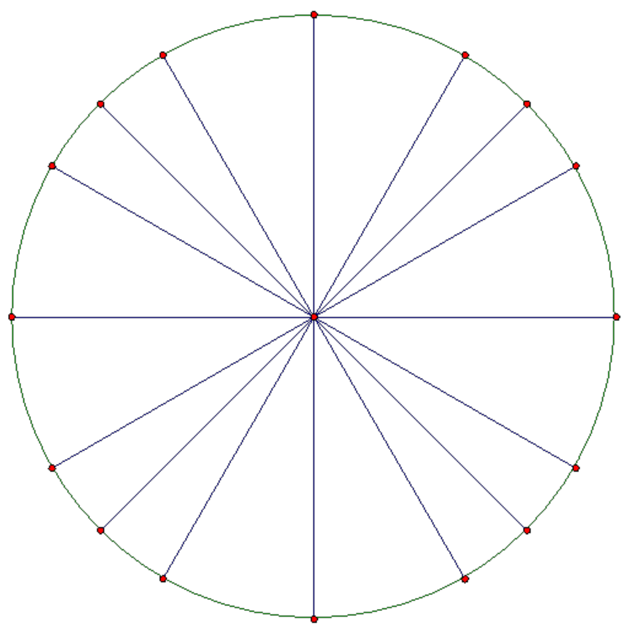
Reference Angle:



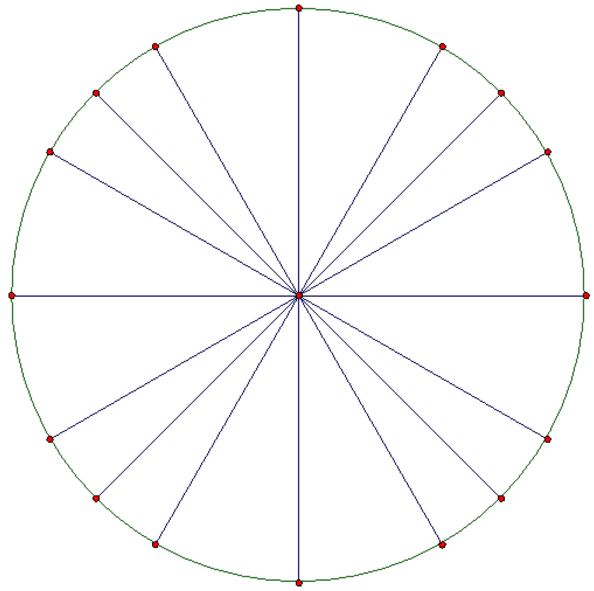
c) $\sin\left(-\frac{7\pi}{6}\right)$ =___________

Quadrant:

Reference Angle:

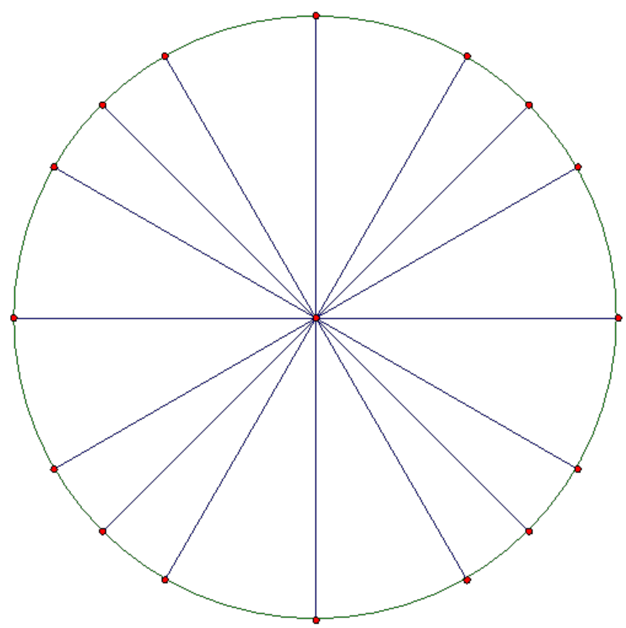


d) $\cos 3\pi =$ _________

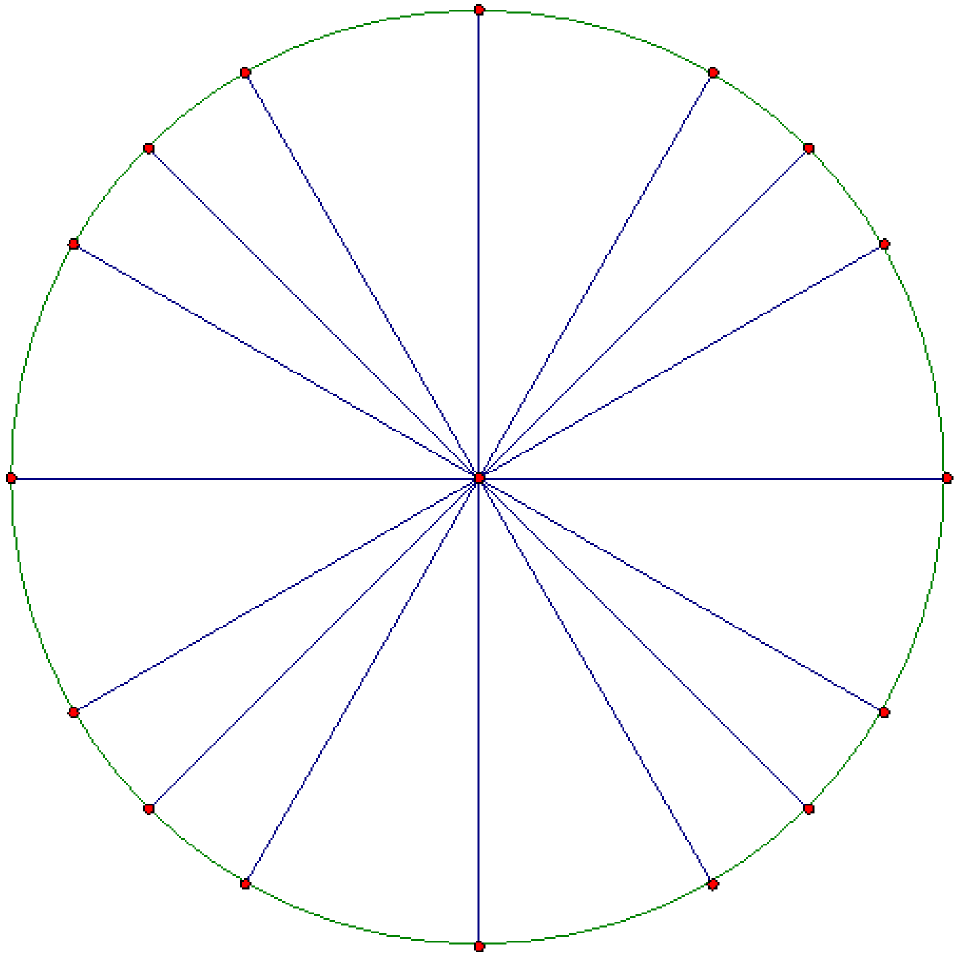


e) $\sin\left(-\frac{14\pi}{3}\right)$ =_________

Quadrant:
Reference Angle:

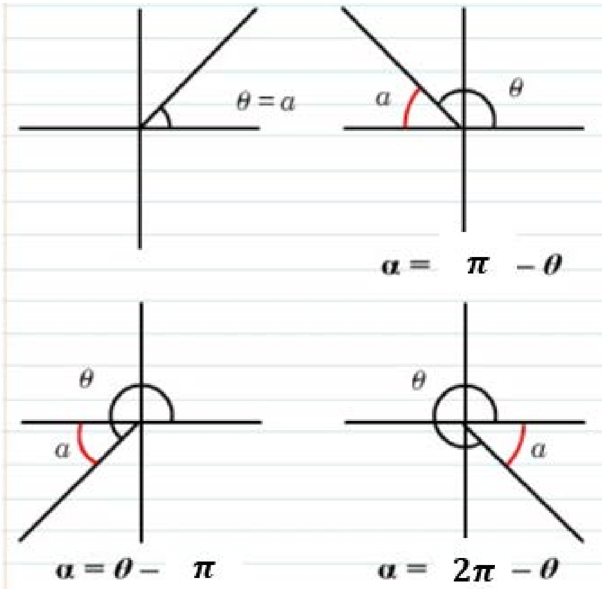


You should be able to label each point on this circle with its exact radian measure. Remember that $\pi = 180°$



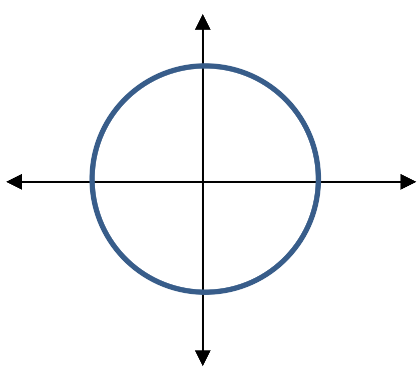
Reference Angles in Radians:

For any angle θ (between 0 and 2π), we can find the reference angle α by using the following table.

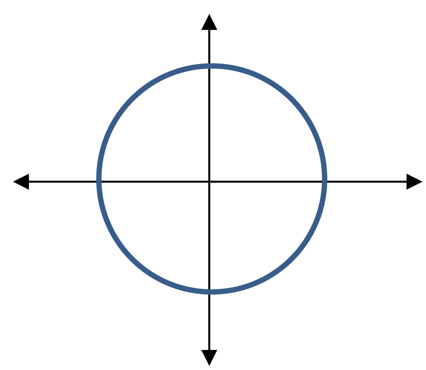


Example: Find the reference angle for the following angles.

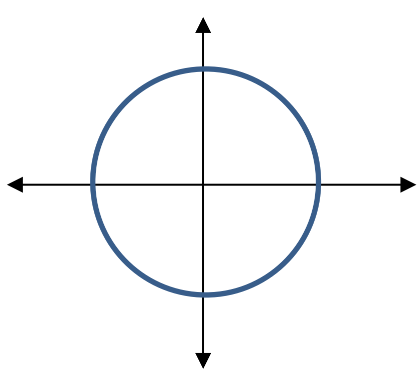
a) $\theta = \frac{7\pi}{6}$



b) $\theta = \frac{5\pi}{3}$



c) $\theta = 2.034443936$



d) $\theta = 5.9012$