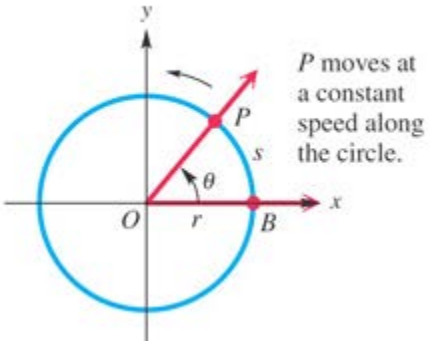
Section 3.4 Linear and Angular Speed



Linear speed: the measure of how fast the position changes of a point along the circle.

$$v = \frac{s}{t}$$

$$linear\ speed = \frac{length\ of\ the\ arc\ traced\ by\ a\ point}{time}$$

Units given as: Feet per sec, cm per sec, yd/min, miles/hour, mph, etc.

Angular speed: the measure of how fast the angle is changing

$$\omega = \frac{\theta}{t},\ \theta\ in\ radians \qquad angular\ speed = \frac{change\ in\ the\ angle}{time}$$

Units given as: Radians per sec, Radians per min, Radians per hour, etc.

It seems logical that there would be a connection between linear speed and angular speed.

$$v = \frac{s}{t}$$

Angular Speed ω	Linear Speed v
$\omega = \frac{\theta}{t}$	$v = \frac{s}{t}$
(ω in radians per unit time t, θ in radians)	$v = \frac{r\theta}{t}$
	$v = r\omega$

How does angular speed affect linear speed?

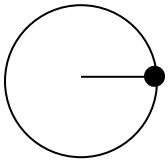
Two rotating disks with the same angular speed $\omega = 2\pi$ radians per hour, will have different linear speeds depending upon the radius.

The disk with the larger the radius will have the larger linear speed because a point on the circle will travel a greater distance in the same amount of time.

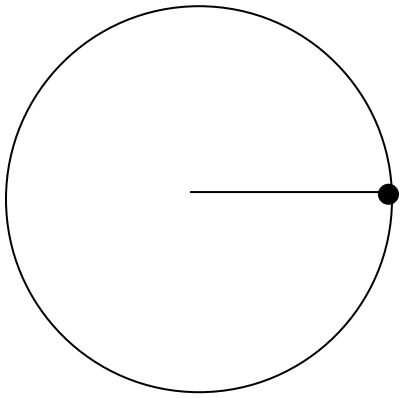
Example: Find the linear speed of each rotating disk, given that $\boldsymbol{v = r\omega}$**.**

Both have same angular speed: $\omega = 2\pi$ radians per hour (1 revolution per hour)

Radius = 1 mile



Radius = 5 miles



Example 1: Use the formula $\boldsymbol{\omega = \frac{\theta}{t}}$ to find the value of the missing variable.

a) $\omega = \frac{2\pi}{3}$ radians per sec, $t = 3$ seconds

b) $\theta = \frac{2\pi}{9}$ radians, $\omega = \frac{5\pi}{27}$ radians per min

Example 2: Use the formula $\boldsymbol{v = r\omega}$ to find the value of the missing variable.

a) $\omega = \frac{2\pi}{3}$ radians per sec, $r = 12$ m

b) $v = 9$ m per sec, $r = 5$ m

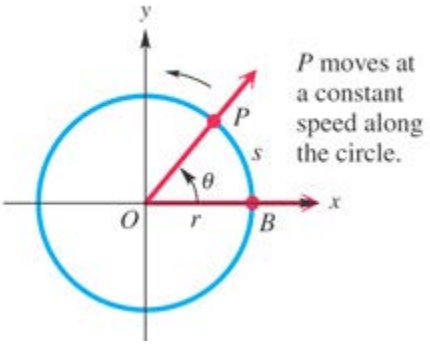
Example 3:

Suppose that *P* is on a circle with radius 15 in., and ray OP is rotating with angular speed $\frac{\pi}{12}$ radians per sec.

a) Find the angle generated by *P* in 10 sec.

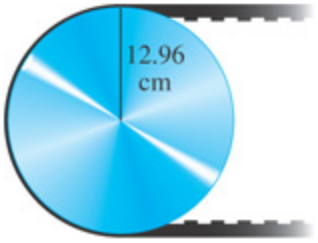
b) Find the distance traveled by *P* along the circle in 10 sec.

c) Find the linear speed of *P* in inches per second.



Example 4:

Speeds of a Pulley and a Belt The pulley shown has a radius of 12.96 cm. Suppose it takes 18 sec for 56 cm of belt to go around the pulley.



(a) Find the linear speed of the belt in centimeters per second.

(b) Find the angular speed of the pulley in radians per second.

Important conversions: 1 revolution = 2π radians 1 ft = 12 in 1 mile = 5280 ft

1 hour = 60 min 1 min = 60 sec

Example 5:

Radius of a Spool of Thread A thread is being pulled off a spool at the rate of 59.4 cm per sec. Find the radius of the spool if it makes 152 revolutions per min.

Example 6:

Speed of a Bicycle The tires of a bicycle have radius 13.0 in. and are turning at the rate of 215 revolutions per min. See the figure. How fast is the bicycle traveling in miles per hour? (*Hint:* 5280 ft = 1 mi)