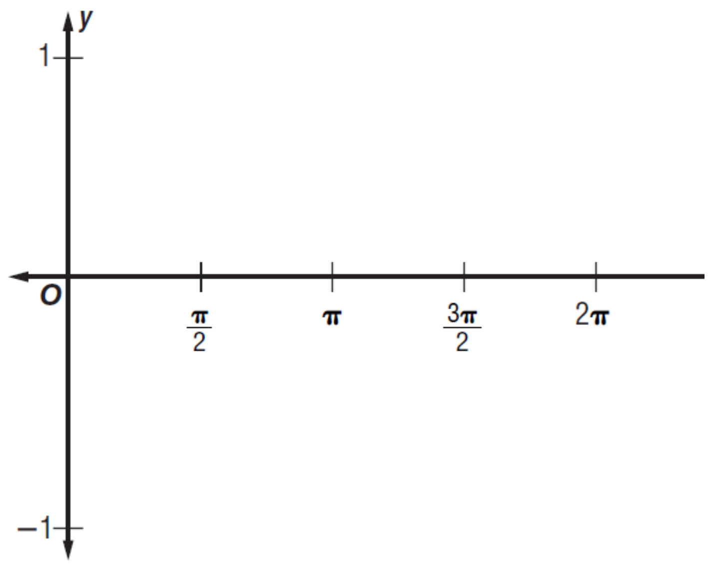
Section 4.1 **Graphs of the Sine and Cosine Functions**

The sine and cosine functions are both called **periodic functions** because their output values repeat themselves. The output values of the basic functions $\boldsymbol{y = \sin x}$ and $\boldsymbol{y = \cos x}$ go through a complete cycle every 2π radians, so they are periodic functions with a **period** of $\boldsymbol{2\pi}$**.**

The graph of the SINE Function $y = \sin x$

Key points on the graph for one period of y = sin x		
x (angle in radians)	$\boldsymbol{y} = \boldsymbol{\sin x}$	**(angle, ratio)**
0	$y = \sin(0) = 0$	$(0,0)$
$\frac{\pi}{2}$	$y = \sin\left(\frac{\pi}{2}\right) = 1$	$\left(\frac{\pi}{2}, 1\right)$
π	$y = \sin(\pi) = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \sin\left(\frac{3\pi}{2}\right) = -1$	$\left(\frac{3\pi}{2}, -1\right)$
2π	$y = \sin(2\pi) = 0$	$(2\pi, 0)$



Definitions:

Period: horizontal length of the interval required to complete one full cycle

Amplitude: vertical distance from the middle of the graph to the top and from the middle to the bottom.

OR

half the difference between the maximum and minimum values.

Summary of $\boldsymbol{y = \sin x}$

Domain: $(-\infty, \infty)$

Range: $[\boldsymbol{-1}, \boldsymbol{1}]$

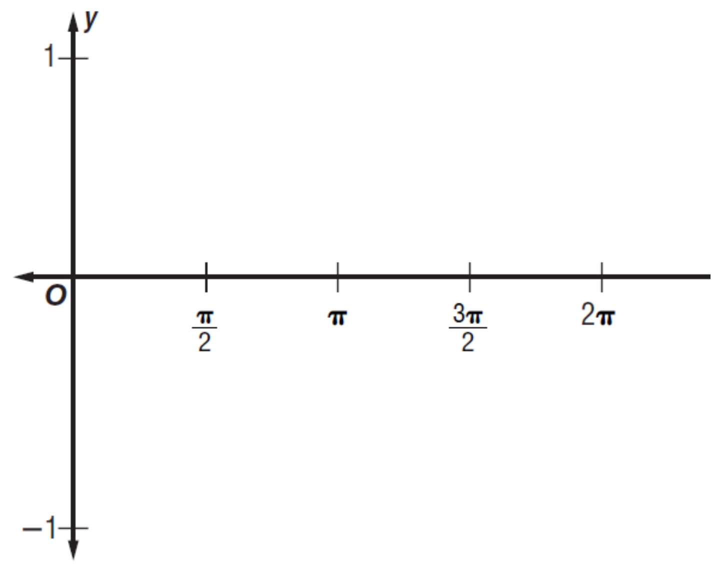
Period: $\boldsymbol{2\pi}$

Amplitude: **1**

The graph is symmetric with respect to the origin, so the function is odd.
By definition of an odd function, $\boldsymbol{\sin(-x) = -\sin x}$**.**

The graph of the COSINE Function

Key points on the graph for one period of y = cos x		
x (angle in radians)	$\boldsymbol{y = \cos x}$	**(angle, ratio)**
0	$y = \cos(0) = 1$	$(0,1)$
$\frac{\pi}{2}$	$y = \cos\left(\frac{\pi}{2}\right) = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = \cos(\pi) = -1$	$(\pi, -1)$
$\frac{3\pi}{2}$	$y = \cos\left(\frac{3\pi}{2}\right) = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = \cos(2\pi) = 1$	$(2\pi, 1)$



Summary of $\boldsymbol{y = \cos x}$

Domain: $(-\infty, \infty)$

Range: $[\mathbf{-1}, \mathbf{1}]$

Period: $\boldsymbol{2\pi}$

Amplitude: **1**

The graph is symmetric with respect to the y-axis, so the function is even.
By definition of an even function, $\boldsymbol{\cos(-x) = \cos x}$**.**

Graphing $y = \boldsymbol{a} \sin x$ or $y = \boldsymbol{a} \cos x$

"a" for us will represent the value that is multiplied times the outside of the function. If "a" is negative, the basic shape will flip vertically.

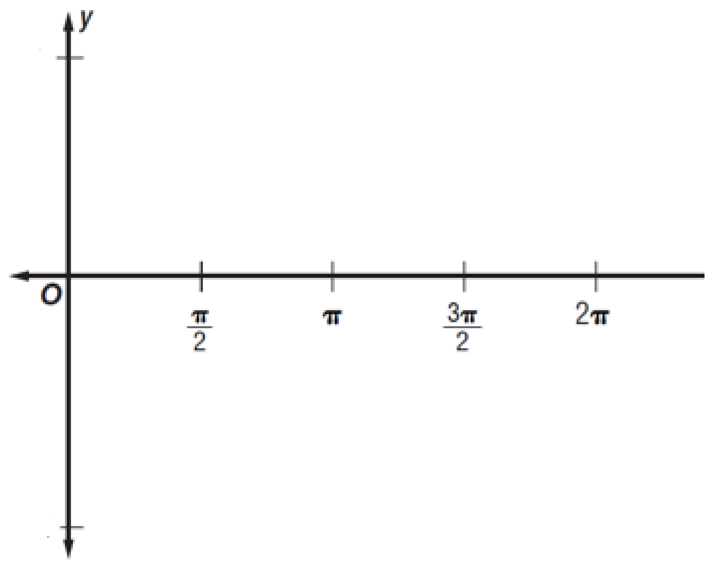
Amplitude of a periodic function: vertical distance from the middle of the graph to the top and from the middle to the bottom.

For the graph of $y = \boldsymbol{a} \sin x$ or $y = \boldsymbol{a} \cos x$ the **amplitude will be $|\boldsymbol{a}|$.**

Amplitude is always considered to be positive!

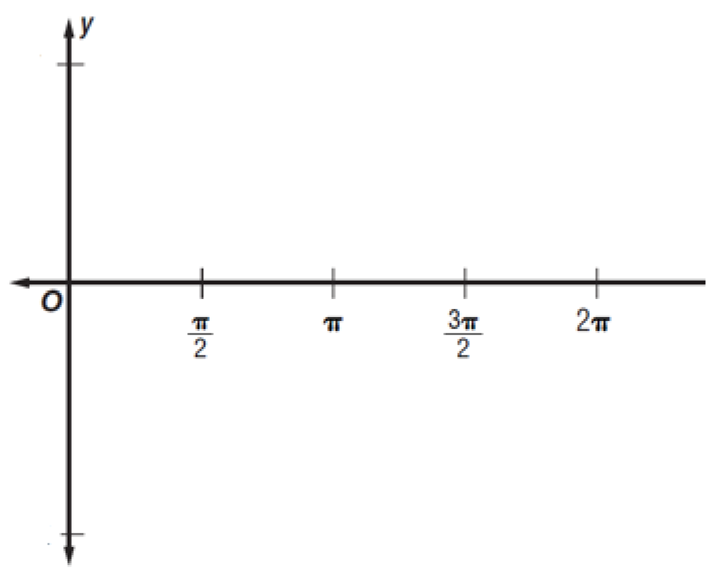
Graph $\boldsymbol{y = 2 \sin x}$ over a one-period interval.

x (angle in radians)	$\boldsymbol{y = 2 * \sin x}$	**(x, y)**
0	$y = 2\sin(0) = 2 * 0$	$(0,0)$
$\frac{\pi}{2}$	$y = 2\sin\left(\frac{\pi}{2}\right) = 2 * 1$	$\left(\frac{\pi}{2}, 2\right)$
π	$y = 2\sin(\pi) = 2 * 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 2\sin\left(\frac{3\pi}{2}\right) = 2 * (-1)$	$\left(\frac{3\pi}{2}, -2\right)$
2π	$y = 2\sin(2\pi) = 2 * 0$	$(2\pi, 0)$



Graph $\boldsymbol{y = -3 \cos x}$ over a one-period interval.

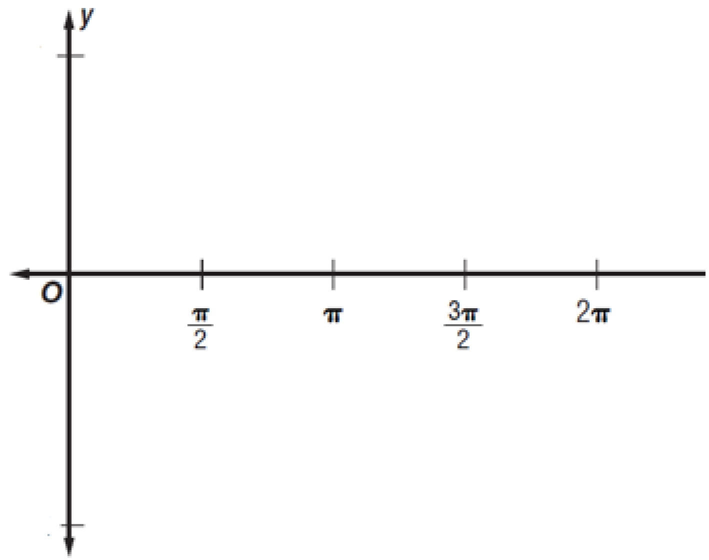
x (angle in radians)	$\boldsymbol{y = -3 * \cos x}$	**(x, y)**
0	$y = -3\cos(0) = -3 * 1$	$(0, -3)$
$\frac{\pi}{2}$	$y = -3\cos\left(\frac{\pi}{2}\right) = -3 * 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = -3\cos(\pi) = -3 * (-1)$	$(\pi, 3)$
$\frac{3\pi}{2}$	$y = -3\cos\left(\frac{3\pi}{2}\right) = -3 * 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = -3\cos(2\pi) = -3 * 1$	$(2\pi, -3)$



Changes made to the outside of the function affect the graph vertically. In other words, it changes the y-values used for the graph.

Example 1: Graph $y = 4\cos x$ over a one-period interval.

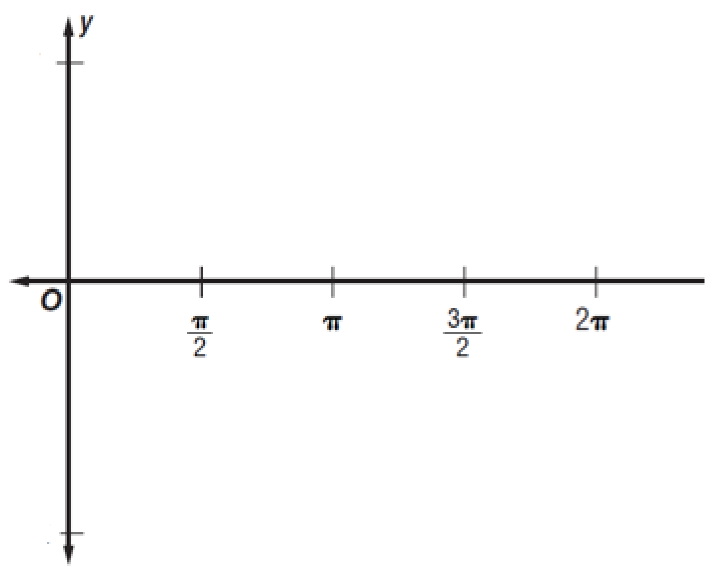
Amplitude:



Period:

Example 2: Graph $\boldsymbol{y = -\frac{2}{3}\sin x}$ over a one-period interval.

Amplitude:



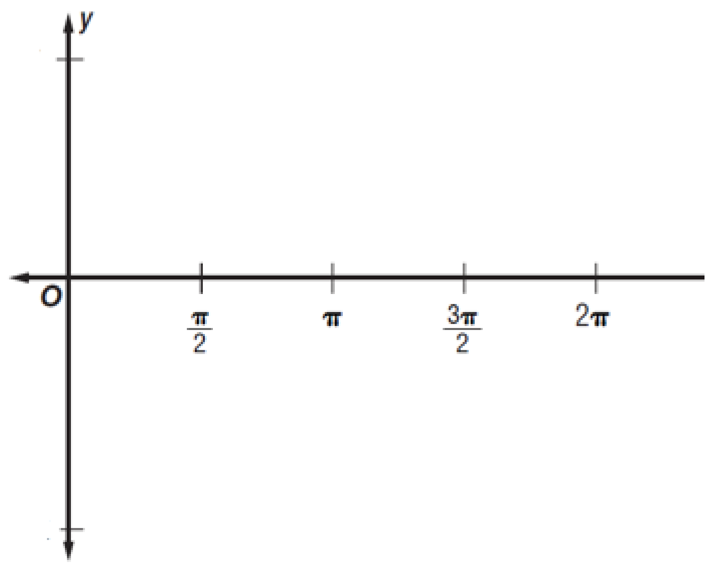
Period:

Graphing $y = \sin \boldsymbol{b}x$ or $y = \cos \boldsymbol{b}x$

The coefficient "b" represents the number of times sine and cosine will complete a cycle between 0 to 2π.

Graph $\boldsymbol{y = \sin 2x}$ over a two-period interval.

x (angle in radians)	$\boldsymbol{y} = \mathbf{sin}(\mathbf{2} * \boldsymbol{x})$	**(x, y)**
0	$y = \sin(2 * 0) = \sin(0) = 0$	$(0,0)$
$\frac{\pi}{4}$	$y = \sin\left(2 * \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2}\right) = 1$	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{2}$	$y = \sin\left(2 * \frac{\pi}{2}\right) = \sin(\pi) = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2 * \frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$	$\left(\frac{3\pi}{4}, -1\right)$
π	$y = \sin(2 * \pi) = \sin(2\pi) = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = \sin\left(2 * \frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{2}\right) = 1$	$\left(\frac{5\pi}{4}, 1\right)$
$\frac{3\pi}{2}$	$y = \sin\left(2 * \frac{3\pi}{2}\right) = \sin(3\pi) = 0$	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{7\pi}{4}$	$y = \sin\left(2 * \frac{7\pi}{4}\right) = \sin\left(\frac{7\pi}{2}\right) = -1$	$\left(\frac{7\pi}{4}, -1\right)$
2π	$y = \sin(2 * 2\pi) = \sin(4\pi) = 0$	$(2\pi, 0)$



If we change the coefficient of *x* that will change the period of the function.

The input angle now is "*bx*."

Since, the coefficient "b" represents the number of times sine and cosine will complete a cycle between 0 to 2π,

The period of any sine or cosine function is $\boldsymbol{\frac{2\pi}{b}}$

Changes made to the inside of the function affect the graph horizontally. In other words, it changes the x-values used for the graph.

To find the 5 x-values that will be used for 1 period,

Step 1. **Determine the period $\boldsymbol{\frac{2\pi}{b}}$.** (Recall that is the required horizontal distance to complete one cycle.)

Step 2. **Find the equal distance between the 5 x-values**. Divide the period by 4. (Recall that the cycles are always divided into 4 equal parts.)

Step 3. **Find all 5 of the x-values used as key points for the graph.** Determine the starting x-value of the graph. Add the value from Step 2 to the starting x-value. Continue adding the value found in Step 2 to x-value until you have found all 5 x-values of the cycle.

Example 1: Graph $\boldsymbol{y = \cos\ 4x}$ over a one-period interval.

Amplitude:

Period:

equal distance between the 5 x-values:

Example 2: Graph $\boldsymbol{y = \sin\ \frac{1}{3}x}$ over a two-period interval.

Amplitude:

Period:

equal distance between the 5 x-values:

Graphing with Change in Amplitude and Change in Period

Graphing $y = \boldsymbol{a} \sin \boldsymbol{b}x$ or $y = \boldsymbol{a} \cos \boldsymbol{b}x$

1) **Find all 5 x-values used as key points for the graph. (**Divide the period by 4. Add that distance to the starting x-value, then continue adding that distance to each x-value until all 5 of the x-values are found.)

2) **Draw in the basic shape** for sine or cosine **using the amplitude** to determine the vertical distance from the middle of the graph.

Example 1: Graph $\boldsymbol{y = 4 \sin 3x}$ over a one-period interval.

Amplitude:

Period:

equal distance between the 5 x-values:

Example 2: Graph $\boldsymbol{y = \frac{1}{2} \cos (-4x)}$ over a two-period interval.

Amplitude:

Period:

equal distance between the 5 x-values:

Example 3: Graph $\boldsymbol{y = -5\cos\frac{1}{2}x}$ over a one-period interval.

Amplitude:

Period:

equal distance between the 5 x-values:

Example 4: Graph $\boldsymbol{y = \frac{2}{3}\sin\left(-\frac{\pi}{4}x\right)}$ over a two-period interval.

Amplitude:

Period:

equal distance between the 5 x-values: