Section 4.3 Graphs of the Tangent and Cotangent Functions

The output values of the basic functions $\boldsymbol{y = \tan x}$ and $\boldsymbol{y = \cot x}$ go through a complete cycle every π radians, so they are periodic functions with **period** $\boldsymbol{\pi}$**.**

The graph of the TANGENT Function

Summary of $\boldsymbol{y = \tan x}$

Domain: $\{\boldsymbol{x} | \boldsymbol{x} \neq (\mathbf{2}\boldsymbol{n}+\mathbf{1})\frac{\boldsymbol{\pi}}{\mathbf{2}}, \boldsymbol{where\ n\ is\ any\ integer}\}$

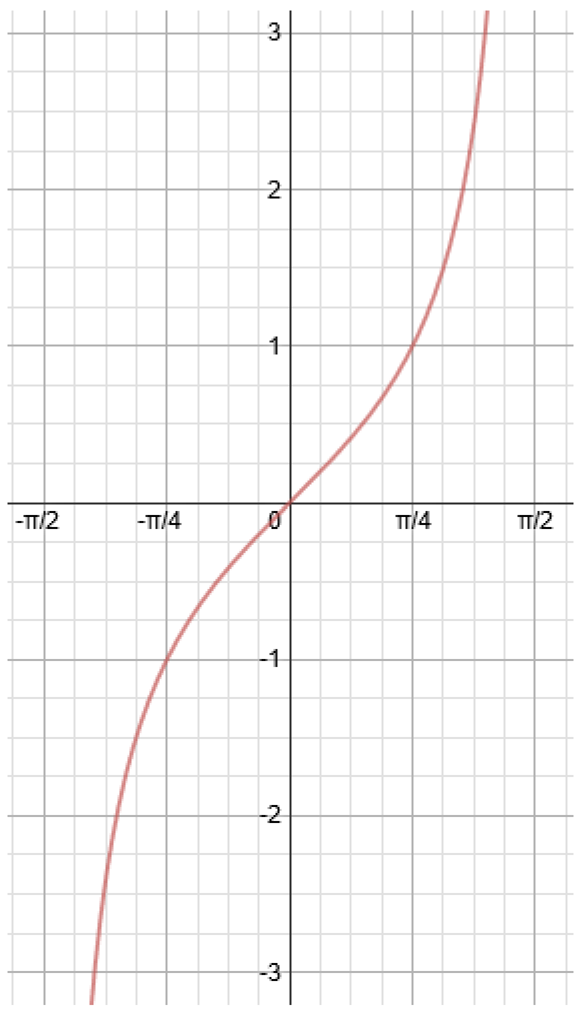
Range: $(-\infty, \infty)$

Period: $\boldsymbol{\pi}$

Vertical Asymptotes: at x-values of the form $\boldsymbol{x} = (\mathbf{2}\boldsymbol{n}+\mathbf{1})\frac{\boldsymbol{\pi}}{\mathbf{2}}$ (at all ODD multiples of $\frac{\boldsymbol{\pi}}{\mathbf{2}}$)

The graph is symmetric with respect to the origin, so the function is odd.
By definition of an odd function, $\mathbf{tan}(-\boldsymbol{x}) = -\mathbf{tan}\,\boldsymbol{x}$**.**

x (angle in radians)	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \sin x$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
$y = \cos x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\boldsymbol{y = \tan x}$					



The graph of the COTANGENT Function

Summary of $\boldsymbol{y = \cot x}$

Domain: $\{\boldsymbol{x | x \neq n\pi, where\ n\ is\ any\ integer}\}$

Range: $(-\infty, \infty)$

Period: $\boldsymbol{\pi}$

Vertical Asymptotes: at x-values of the form $\boldsymbol{x = n\pi}$ (at all multiples of)

The graph is symmetric with respect to the origin, so the function is odd.
By definition of an odd function, $\boldsymbol{\cot(-x) = -\cot x}$**.**

x (angle in radians)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y = \cos x$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
$y = \sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\boldsymbol{y = \cot x}$					

Graphing $\boldsymbol{y = c + a\tan[b(x-d)]}$ $\qquad\qquad$ $\boldsymbol{y = c + a\cot[b(x-d)]}$

1) <u>Determine the **x-values**</u> of the asymptotes <u>at the **start and end**</u> of the period: Set up a compound inequality by placing the argument between the original start and end points. Solve the inequality for x.

-For tangent: $-\frac{\pi}{2} < agrument < \frac{\pi}{2}$ $\qquad$ -For cotangent: $0 < agrument < \pi$

2) <u>Determine the remaining **x-values**</u>**.** Determine the period $\boldsymbol{\frac{\pi}{b}}$ **.** Divide by 4. Add the equal distance to the starting x-value found in step 1.

3) "c" Vertical Shift: Draw a dotted horizontal line for the new middle.

4) "a" Stretch the graph vertically from the middle "a" units. If "a" is negative, the graph flips vertically. (NOTE: Tangent and Cotangent do not have an "amplitude")

5) Draw the basic shape of tangent or cotangent within the asymptotes.

Graph each of the following over a one period interval.

a) $y = \tan\frac{1}{2}x$

Period: $\boldsymbol{\frac{\pi}{b}}$

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b) $y = 2\cot x$

Period: $\boldsymbol{\frac{\pi}{b}}$

c) $y = 1 - 2\tan x$

Period:

d) $y = \cot\left(3x + \frac{\pi}{4}\right)$

Period: