Section 5.3 Sum and Difference Identities for Cosine

$$\boldsymbol{\cos(A+B) = \cos A \cos B - \sin A \sin B}$$

$$\boldsymbol{\cos(A-B) = \cos A \cos B + \sin A \sin B}$$

NOTE: Angles A and B can be measured in degrees or radians.

We know the exact trigonometric function values of several angles on the unit circle.

$0°, 30°, 45°, 60°, 90°, etc.$ $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, etc.$ By forming the sums and differences of these angles, we can find exact function values of angles that are not commonly known.

For example, we know the exact function values for $30°$ and $45°$. Using these identities we can find the exact cosine value of their sum and of their difference.

Sum

$\boldsymbol{\cos(30° + 45°) =}$

Difference

$\boldsymbol{\cos(30° - 45°) =}$

Example 1: Find the *exact* value of each expression.

a) $\cos 195°$

b) $\cos\left(-\frac{\pi}{12}\right)$

c) $\cos 173° \cos 128° + \sin 173° \sin 128°$

d) $\cos\frac{\pi}{18}\cos\frac{\pi}{9} - \sin\frac{\pi}{18}\sin\frac{\pi}{9}$

Example 2: Given information about two different angles s and t, we can find function values of their sum $s+t$ or difference $s-t$.

a) Suppose that $\sin s = \frac{3}{5}$ and $\cos t = -\frac{12}{13}$. If both s and t are in quadrant II, find $\cos(s+t)$.

b) Suppose that $\sin s = \frac{2}{3}$ and $\sin t = -\frac{1}{3}$. If both s is in quadrant II and t is in quadrant IV, find $\cos(s-t)$.