Section 6.3 Trigonometric Equations PART 2

In this section, we discuss trigonometric equations that involve functions of half-angles and multiples of angles. Solving these equations often requires adjusting solution intervals to fit given domains.

NOTE: $\frac{\cos 2\theta}{2} \neq \cos\theta$

When solving an equation with $n\theta$ as the input angle, we need to use adjusted solution intervals or identities.

How to solve equations for θ if given multiples of angles, $n\theta$

1. **Isolate** $\cos n\theta$, $\sin n\theta$, etc.

2. **Solve for $n\theta$** within the adjusted solution interval. Multiply each part of the original inequality by n.

3. **Divide by n** to solve for θ.

<u>Adjusting solution intervals to fit given domains:</u>

In order to solve for θ, $0° \leq \theta < 360°$,
we must first solve for $n\theta$ over the interval $n * 0° \leq n\theta < n * 360°$.

If the equation contains an input angle of $\mathbf{2\theta}$

and we are asked to solve the equation for θ over the original interval $[0°, 360°)$ $0° \leq \theta < 360°$,

then we must first solve for 2θ over the interval $[0°, 720°)$ also written as $0° \leq 2\theta < 720°$.

If the equation contains an input angle of $\mathbf{\frac{1}{2}\theta}$ **or** $\mathbf{\frac{\theta}{2}}$

and we are asked to solve the equation for θ over the original interval $[0°, 360°)$ $0° \leq \theta < 360°$,

then we must first solve for $\frac{1}{2}\theta$ or $\frac{\theta}{2}$ over the interval $[0°, 180°)$ also written as $0° \leq \frac{\theta}{2} < 180°$

If the equation contains an input angle of $\mathbf{3\theta}$

and we are asked to solve the equation for θ over the original interval $[0°, 360°)$ $0° \leq \theta < 360°$,

then we must first solve for 3θ over the interval ______________________________________

Example 1: Solve for θ: $\cos 2\theta = \frac{\sqrt{3}}{2}$ over the interval $[0°, 360°)$

1. **Isolate** $\cos n\theta$.

2. **Solve for $\boldsymbol{n\theta}$** within the adjusted solution interval of ______________________________

3. **Divide by $\boldsymbol{n}$** to solve for θ.

Let's prove that these solutions really do work in the original equation, $\cos 2\theta = \frac{\sqrt{3}}{2}$ over the interval $[0°, 360°)$.

$\theta = 15°,\ 165°,\ 195°,\ 345°$.

Example 2: Solve for x: $3\tan 3x = \sqrt{3}$ over the interval $[0, 2\pi)$

1. Isolate ____________________

2. Solve for _______ with adjusted interval of ___________________________

3. Divide by 3. (NOTE: Dividing by 3 is the same as multiplying by the reciprocal $\frac{1}{3}$)

Example 3: Solve $\sqrt{2}\sin 3x - 1 = 0$ for all solutions in radians.

1. Isolate ____________________

2. Solve for <u>all solutions</u> in radians. (NOTE: Since we are already asked to find all coterminal angles, we don't adjusting solution intervals)

3. Divide all terms by 3. (NOTE: Dividing by 3 is the same as multiplying by the reciprocal $\frac{1}{3}$)

Example 4: Solve $2\sin\frac{x}{2} = 1$ over the interval $[0, 2\pi)$

1. Isolate ____________________

2. **Solve for $\boldsymbol{nx}$** within the adjusted solution interval of ______________________________________

3. **Divide by $\boldsymbol{n}$** to solve for x. (NOTE: Dividing by $\frac{1}{2}$ is the same as multiplying by the reciprocal $\frac{2}{1}$)

Example 5: Solve $\cos\frac{\theta}{2} = \sqrt{2} - \cos\frac{\theta}{2}$ over the interval $[0, 360°)$

Example 6: Solve $2\sqrt{3}\sin\frac{x}{2} = 3$ for all solutions.

Example 7: Solve $2 - \sin 2\theta = 4\sin 2\theta$ for all solutions.

Example 8: Solve $2\cos^2 2\theta = 1 - \cos 2\theta$ for all solutions.

Using Double Angle Identities

Example 9: Solve $\sin 2\theta = \sin\theta$ over the interval $[0, 360°)$

Example 10: Solve $\cos 2x = 1 - \sin x$ over the interval $[0, 2\pi)$