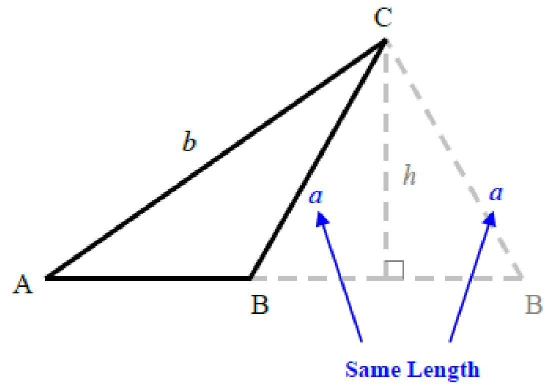
Section 7.2 The Ambiguous Case of the Law of Sine

When we consider, SSA, we know two sides, but only one angle is known (and this angle is NOT between the given two sides.) This introduces ambiguity about what the other 2 angles could be and, in turn, what the remaining side might be. (Recall: There is no SSA congruence axiom.)

For example, if we know the measure of angle *A* and the lengths of sides *b* and *a*, we see that side *a* could swing in or out to create two different triangles. This could make side *c* to be longer or shorter.



The ambiguous case results in zero, one, or two triangles.

Number of Triangles Satisfying the Ambiguous Case (SSA)

Let sides *a* and *b* and angle *A* be given in a triangle ABC. (The law of sines can be used to calculate the value of *B*.)

1. **No triangle:** If applying the law of sines results in an equation having $\sin B > 1$, then no such angle *B* would exist. Therefore, no triangle satisfies the given conditions.

2. **One triangle (no possibility of 2)**: If $\sin B = 1$, then $B = 90°$. Therefore, only one triangle exists under the given conditions. It is a right triangle.

3. **Either one or two triangles:** If $0 < \sin B < 1$, then there could be two possible values for *B*. Angle *B* could be than angle from quadrant 1, $\boldsymbol{B_1 = B}$ or from quadrant 2, $\boldsymbol{B_2 = 180° - B}$. Therefore, there could be either one or two triangles.

a) **1st triangle:** Angle *A* is given, use the angle *B* from quadrant 1, $B_1 = B$,
then $C_1 = 180° - A - B_1$.

Then angles A, B_1, and C_1 will be the angles for the first triangle.

b) **Possible 2nd triangle**: Angle *A* is given, use the angle *B* from quadrant 2, $B_2 = 180° - B$.
If $A + B_2 < 180°$, then a second triangle can be formed and $C_2 = 180° - A - B_2$.

Then angles A, B_2, and C_2 will be the angles for the second triangle.

No triangle possible

Example 1: Solve triangle ABC if $= 75°30'$, $a = 17.9$ cm, and $c = 13.2$ cm.

Possibility of either one or two triangles

Example 2: Solve triangle ABC if $A = 61.4°$, $a = 35.5$ cm, and $b = 39.2$ cm.

Possibility of either one or two triangles

Example 3: Solve triangle ABC if $B = 68.7°$, $b = 25.4$ in, and $a = 19.6$ in.