Section 7.3 The Law of Cosines

A few facts about triangles that we need to keep in mind.

1) A triangle can only have 1 obtuse angle.

2) If a triangle has 1 obtuse angle, then it is the largest angle in the triangle.

3) The longest side is opposite of the largest angle in the triangle.

4) The smallest side is opposite of the smallest angle in the triangle.

5) The sum of the lengths of any two sides must be greater than the length of the remaining side.

Question 1: Is the following information valid for a triangle ABC?

$A = 104°$, $a = 26.8$ m, and $b = 31.3$m

Question 2: Is the following information valid for a triangle ABC?

$a = 3$ in, $b = 4$ in , and $c = 10$ in

Question 3: Given the following information, which angle could possibly be obtuse? A, B, or C?

$a = 13.9$ m, $b = 9.8$ m , and $c = 11.3$ m

Question 4: Given the following information, which angle could possibly be obtuse? A, B, or C?

$A = 41.4°$, $b = 6.7$ cm, and $c = 13.8$ cm

When solving oblique triangles SSS and SAS, there is not enough information given to solve using the law of sines.

For example, consider the case of SSS. **Solve triangle ABC if $a = 11.3$ m , $b = 12.9$ m , $c = 15.4$ m.**

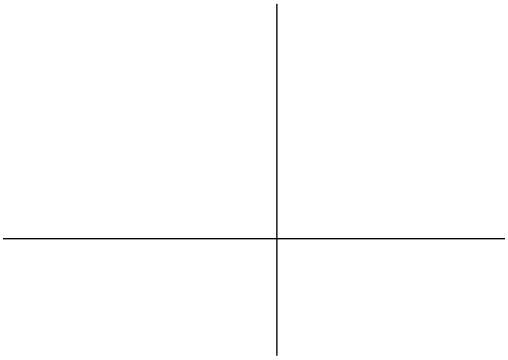
We need another law to solve SSS and SAS. For these, we will need the Law of Cosines.

The Law of Cosines

In any triangle ABC, with sides a, b, c, the following three identities will hold.

$a^2 = b^2 + c^2 - 2bc\cos A$, $b^2 = a^2 + c^2 - 2ac\cos B$, $c^2 = a^2 + b^2 - 2ab\cos C$

Where does it come from?



A Summary of steps need when solving Oblique Triangles

If you are given this information:	Suggested Procedure for Solving:
SAA (AAS) or ASA LAW OF SINES	**Step 1:** Find the remaining angle using the angle sum formula for a triangle, $A + B + C = 180°$ **Step 2:** Use the law of sines to find the remaining sides.
SSA (ASS ☺) LAW OF SINES	*This is the ambiguous case. There may be no triangle, one triangle, or two triangles.* **Step 1:** Find either unknown angle using the law of sines. If no such angle exists, then no such triangle exists. **Step 2:** If the Quadrant I angle exists, also find the Quadrant II angle. **Step 3:** Find the remaining angles for both of the possible triangles using the angle sum formula for a triangle, $A + B + C = 180°$. Determine if 1 or 2 valid triangles exists. **Step 4:** Find the remaining side of each valid triangle by using the law of sines.
SAS LAW OF COSINES	**Step 1:** Find the third side using the law of cosines. **Step 2:** Find the SMALLER of the two remaining angles using the law of sines. (Find the smaller of the two angles, because we know it can NOT be obtuse. This will ensure that here will be no ambiguity about the measure of this angle, since the smaller of the two angles can only be from quadrant I.) **Step 3:** Find the remaining angle using the angle sum formula for a triangle, $A + B + C = 180°$
SSS LAW OF COSINES	**Step 1:** Find the largest angle using the law of cosines. (We find the largest first, because it is the only angle that could possibly be obtuse. Since we are using the law of cosines, there will be no ambiguity about the measure of this angle.) **Step 2:** Find either remaining angle using the law of sines. **Step 3:** Find the remaining angle using the angle sum formula for a triangle, $A + B + C = 180°$

SSS

Step 1: Find the largest angle using the law of cosines. (We find the largest first, because it is the only angle that could possibly be obtuse. Since we are using the law of cosines, there will be no ambiguity about the measure of this angle.)

Step 2: Find either remaining angle using the law of sines.

Step 3: Find the remaining angle using the angle sum formula for a triangle, $A + B + C = 180°$

Example 1: Solve the following triangle.

$a = 9.3$ cm, $b = 5.7$ cm , and $c = 8.2$ cm

Example 2: Solve the following triangle.

$a = 42.9$ m, $b = 37.6$ m , and $c = 62.7$ m

SAS

Step 1: Find the third side using the law of cosines.

Step 2: Find the SMALLER of the two remaining angles using the law of sines.

(Find the smaller of the two angles, because we know it can NOT be obtuse. This will ensure that here will be no ambiguity about the measure of this angle, since the smaller of the two angles can only be from quadrant I.)

Step 3: Find the remaining angle using the angle sum formula for a triangle, $A + B + C = 180°$

Example 3: Solve the following triangle.

$A = 41.4°$, $b = 2.78$ yd, $c = 3.92$ yd

Example 4: Solve the following triangle.

$B = 42.3°$, $a = 12.9$ cm, $c = 15.4$ cm