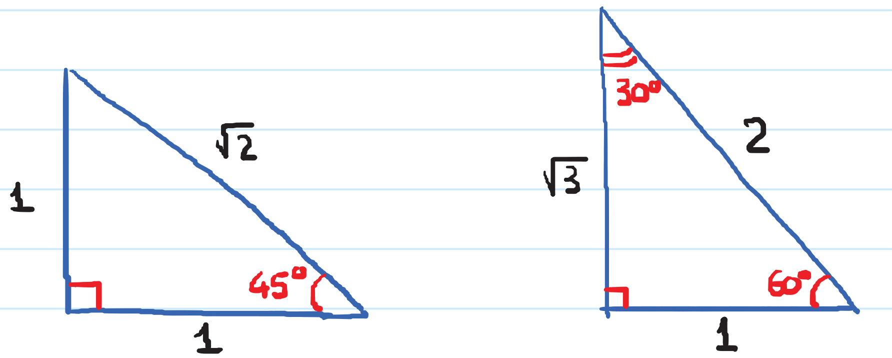
2.2. Trig Function of Non-Acute Angles

Thursday, January 31, 2019 8:03 AM

Reminder:

SOHCAHTOA

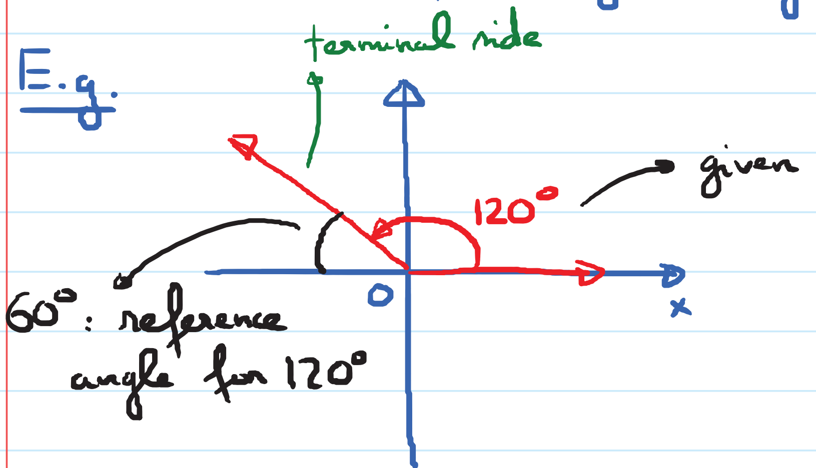


45° - 45° - 90°   30° - 60° - 90°

Reference Angle:

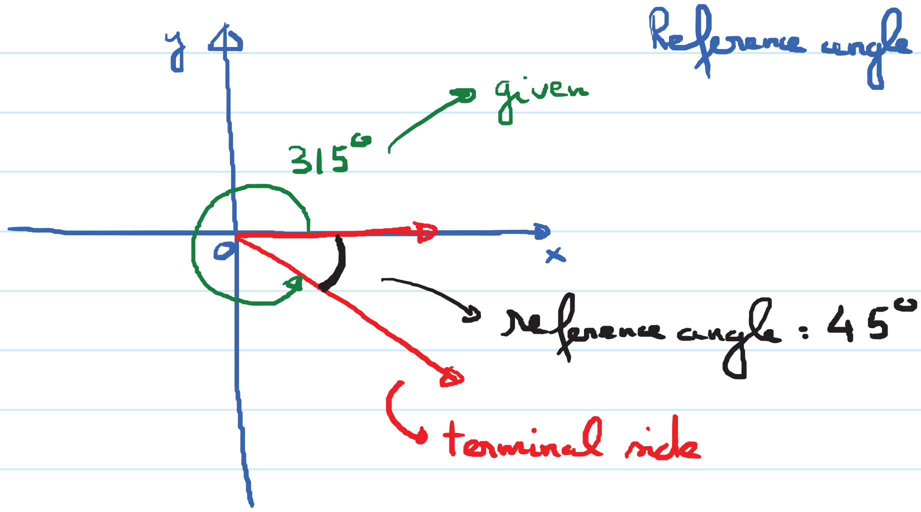
Def: Given an angle, the reference angle for that angle is the positive acute angle made by the terminal side of the given angle and the x-axis.

E.g.



Find the reference angle for this angle

Reference angle of 315°



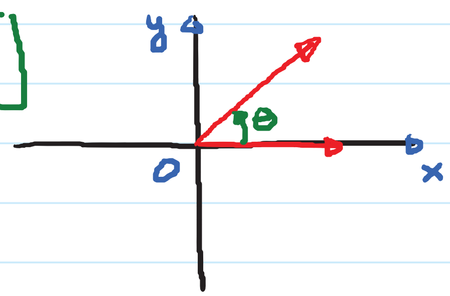
Process for calculating the reference angle of a given angle θ.

Step 1: Turn θ into an angle in between 0° and 360° by adding / subtracting appropriate multiple of 360° to it.

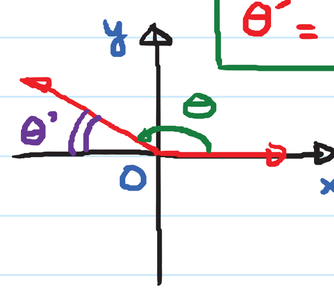
Step 2: Once θ is in between 0° and 360° there are 4 cases:

reference angle

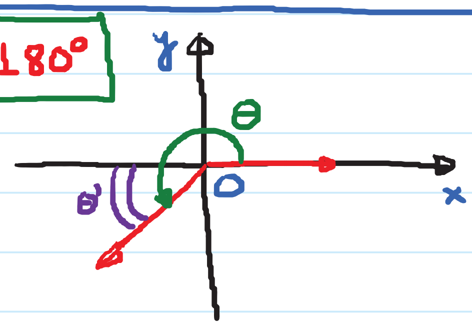
$\boxed{\theta' = \theta}$



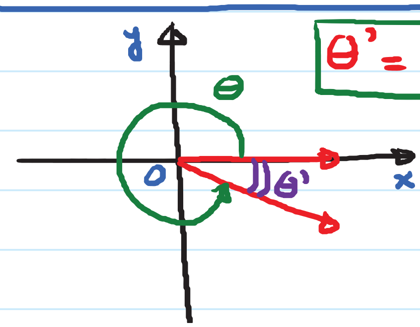
$\boxed{\theta' = 180° - \theta}$



$\boxed{\theta' = \theta - 180°}$



$\boxed{\theta' = 360° - \theta}$



E.g. Find the reference angle for the given angle.

(a) $-290°$

Step 1: $\theta = -290° + 360° = 70°$.

Step 2: Reference angle: $\theta' = \theta = \boxed{70°}$

(b) $1130°$

Step 1: $\theta = 1130° - 3 \cdot 360° = 50°$

Step 2: Reference angle: $\theta' = \theta = \boxed{50°}$

(c) $927°$

Step 1: $\theta = 927° - 2 \cdot 360° = 207°$

Step 2: $\theta' = 207° - 180° = \boxed{27°}$

(d) $-3428°$ $\quad 3428° / 360° \approx 10$

Step 1: $\theta = -3428° + 10 \cdot 360° = 172°$

Step 2: $\theta' = 180° - 172° = \boxed{8°}$

Finding Trig Function Values for Nonquadrantal angle related to the special angles $30°, 45°, 60°$.

Given an angle θ.

Step 1: Find the reference angle θ' of θ.

Step 2: Find the trig function values of the reference angle θ'

Step 3: Determine the correct sign of the trig function values of θ.

E.g. Find the 6 trig function values of $\theta = 240°$

Step 1: Find reference angle θ' of θ

$\theta' = 60°$

Step 2: Find $\sin\theta'$, $\cos\theta'$, $\tan\theta'$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \sqrt{3}$

Step 3: Correct the signs.

θ is in QIII: $\sin 240° = -\frac{\sqrt{3}}{2}$; $\cos 240° = -\frac{1}{2}$

$\tan 240° = \sqrt{3}$; take reciprocals . . .

E.g. 150°

Step 1: Reference angle:

$$\theta' = 180^\circ - 150^\circ = 30^\circ$$

Step 2: Find $\sin\theta'$, $\cos\theta'$, $\tan\theta'$

$$\sin 30^\circ = \frac{1}{2}\ ;\ \cos 30^\circ = \frac{\sqrt{3}}{2}\ ;\ \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Step 3: Fix the sign.

$$\sin 150^\circ = \frac{1}{2}\ ;\ \cos 150^\circ = -\frac{\sqrt{3}}{2}$$

$$\tan 150^\circ = -\frac{\sqrt{3}}{3}$$

E.g. 315°

Step 1: $\theta' = 360^\circ - 315^\circ = 45^\circ$

Step 2: $\sin 45^\circ = \frac{\sqrt{2}}{2}$; $\cos 45^\circ = \frac{\sqrt{2}}{2}$; $\tan 45^\circ = 1$

Step 3: $\sin 315^\circ = -\frac{\sqrt{2}}{2}$; $\cos 315^\circ = \frac{\sqrt{2}}{2}$

$$\tan 315^\circ = -1.$$

E.g. Find the exact value of the given expression.

(a) $\sin(-150°)$ (b) $\cot 1035°$

(c) $\cos(-300°)$ (d) $\sec(750°)$

(d) $\sec(750°)$

Step 1: Turn θ into a coterminal angle in between $0°$ and $360°$

$$\theta = 750° - 2 \cdot 360° = 30°$$

Step 2: Reference angle

$$\theta' = \theta = 30°$$

Step 3: $\sec 30° = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$.

Step 4: Correct sign: $750°$ is in QI.

$$\boxed{\sec 750° = \frac{2\sqrt{3}}{3}}$$

(a) (1) $\theta = -150° + 360° = 210°$

(2) $\theta' = 210° - 180° = 30°$

(3) $\sin\theta' = \sin 30° = \dfrac{1}{2}$

(4) θ is in Q III, $\sin\theta < 0$

$$\boxed{\sin(-150°) = -\frac{1}{2}}$$

(b) $\cot 1035^\circ$

(1) $\theta = 1035^\circ - 2 \cdot 360^\circ = 315^\circ$

(2) $\theta' = 360^\circ - 315^\circ = 45^\circ$

(3) $\cot 45^\circ = \dfrac{adj}{opp} = 1$

(4) $\cot(1035^\circ) = -1$

(c) $\cos(-300^\circ)$

(1) $\theta = -300^\circ + 360^\circ = 60^\circ$

(2) $\theta' = \theta = 60^\circ$

(3) $\cos 60^\circ = \dfrac{1}{2}$

(4) $\cos(-300^\circ) = \dfrac{1}{2}$