1.3 Trigonometric Functions

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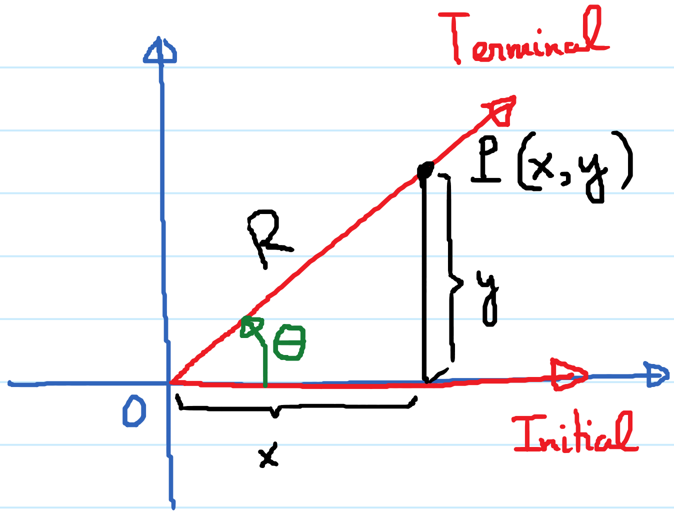


θ : angle in standard position

P : arbitrary point on terminal side of θ.

R = distance from O to P.

Then $\boxed{R = \sqrt{x^2 + y^2}}$

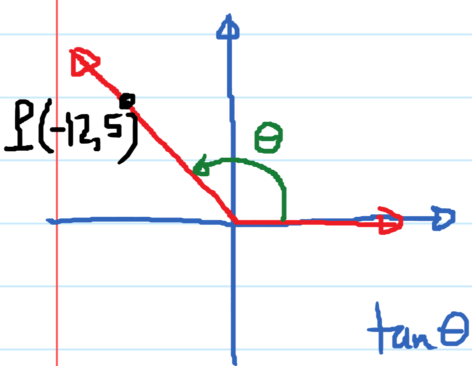
Definition of the 6 basic trig. functions of the angle θ

$\sin\theta := \dfrac{y}{R}$ (sine of theta) ; $\cos\theta := \dfrac{x}{R}$ (cosine) ; $\tan\theta := \dfrac{y}{x}$ $(x \neq 0)$ (tangent)

$\cot\theta := \dfrac{x}{y}$ (cotangent) $(y \neq 0)$; $\sec\theta := \dfrac{R}{x}$ (secant) $(x \neq 0)$; $\csc\theta := \dfrac{R}{y}$ $(y \neq 0)$ (cosecant)

E.g. The terminal side of an angle θ in standard position passes through the point $(-12, 5)$

Q: Find the values of the 6 trig functions of θ.



$R = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (5)^2} = \sqrt{169} = 13$

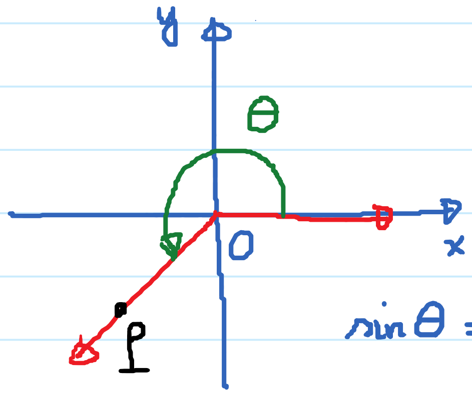
$\sin\theta = \dfrac{5}{13}$; $\cos\theta = \dfrac{-12}{13} = -\dfrac{12}{13}$

$\tan\theta = \dfrac{5}{-12} = -\dfrac{5}{12}$; $\cot\theta = -\dfrac{12}{5}$, $\sec\theta = -\dfrac{13}{12}$

$\csc\theta = \dfrac{13}{5}$

$$\csc\theta = \frac{13}{5}$$

E.g. Same question, point now is $(-2\sqrt{3}, -2)$.



$$R = \sqrt{(-2\sqrt{3})^2 + (-2)^2}$$

$$= \sqrt{12+4} = 4$$

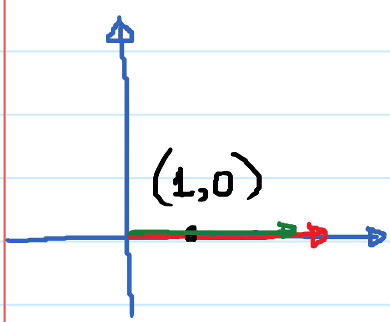
$$\sin\theta = -\frac{1}{2} \; ; \; \cos\theta = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{\cancel{-2}}{\cancel{-2}\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

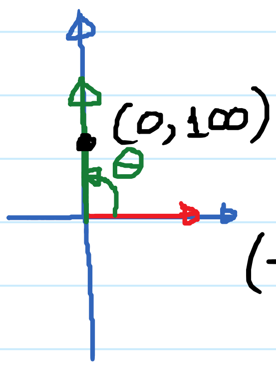
$$\cot\theta = \frac{\cancel{-2}\sqrt{3}}{\cancel{-2}} = \sqrt{3} \; ; \; \sec\theta = \frac{4}{-2\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\csc\theta = -2 \qquad = -\frac{2\sqrt{3}}{3}$$

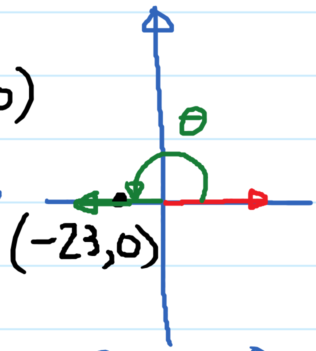
Values of the 6 trig functions of the quadrantal angles



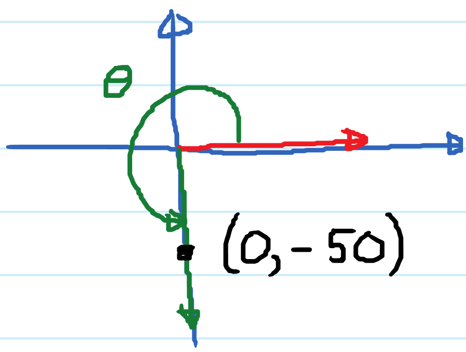
$\theta = 0°$



$\theta = 90°$



$\theta = 180°$



$\theta = 270°$

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θ	Pt on term.	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°	$(1,0)$; $R=1$	0	1	0	undef.	1	undef.
90°	$(0,100)$; $R=100$	1	0	undef	0	undef	1
180°	$(-23,0)$; $R=23$	0	-1	0	undef	-1	undef
270°	$(0,-50)$ $R=50$	-1	0	undef	0	undef	-1

Note: Coterminal angles have the same trig functions values.

$\sec(360^\circ) = \sec(0^\circ) = 1$.

$\sin(-90^\circ) = \sin(270^\circ) = -1$

$\csc(450^\circ) = \csc(90^\circ) = 1$.

We have found the trig functions values of

$\theta + n\cdot 360^\circ$, $n = \dots, -3, -2, -1, 0, 1, 2, 3, \dots$

and θ is one of the angles $0^\circ, 90^\circ, 180^\circ, 270^\circ$.

E.g. Evaluate the given expressions:

$$\sin(\underbrace{-270^\circ}_{\text{coterm. w } 90^\circ}) + 3\cdot\tan(\underbrace{1800^\circ}_{\text{coterm. w } 0^\circ})$$

$$= \underbrace{\sin(90^\circ)}_{1} + 3\cdot\underbrace{\tan(0^\circ)}_{0}$$

$$= 1 + 3\cdot 0 = \boxed{1}$$

E.g.

exponent

$$\sec^{\boxed{2019}}(540^\circ) = \left[\sec(\underbrace{540^\circ}_{\text{coterm. w } 180^\circ})\right]^{2019}$$

$$= \left[\sec(180^\circ)\right]^{2019} = (-1)^{2019} = -1$$

E.g.

$$\cos^2(-180^\circ) - 7\sin^2(-180^\circ)$$

$$= \left[\cos(180^\circ)\right]^2 - 7\left[\sin(180^\circ)\right]^2$$

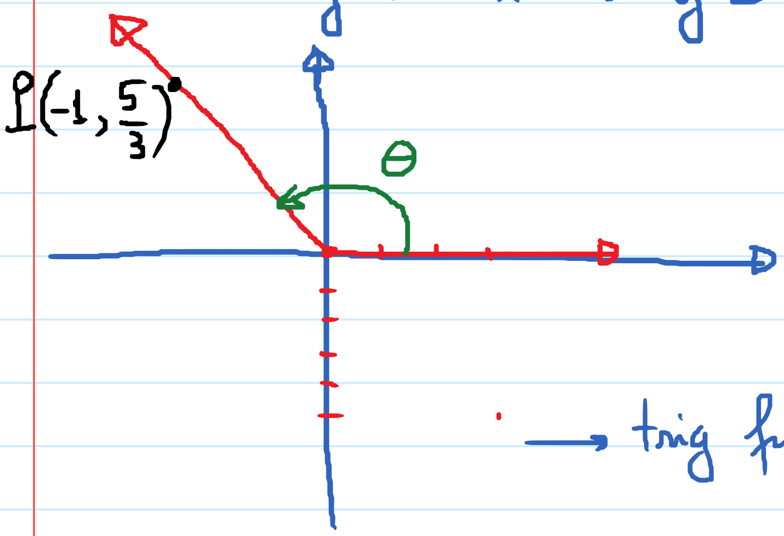
$$= (-1)^2 - 7\cdot(0)^2 = \boxed{1}.$$

E.g. Equation of the terminal side of an angle θ in S.P. is given with a restriction on x:

$$-5x - 3y = 0 \; ; \; x \leq 0$$

Q: Find the trig functions values of θ.

$$-3y = 5x \rightarrow y = \frac{-5}{3}x$$



choose $x = -1$.

$$y = -\frac{5}{3}.(-1) = \frac{5}{3}$$

$R = \ldots$

$\longrightarrow$ trig function values...