1.4. Using the Definitions of Trig Functions

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Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \; ; \; \cos\theta = \frac{1}{\sec\theta} \; ; \; \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \; ; \; \sec\theta = \frac{1}{\cos\theta} \; ; \; \cot\theta = \frac{1}{\tan\theta}$$

E.g. Given: $\cot\theta = -4$. Find $\tan\theta$.

$$\boxed{\tan\theta = -\frac{1}{4}}$$

E.g. Given: $\cos\theta = -\frac{2}{\sqrt{20}}$. Find $\sec\theta$

$$\sec\theta = -\frac{\sqrt{20}}{2} = -\frac{\cancel{2}\sqrt{5}}{\cancel{2}} = -\sqrt{5}$$

$$\boxed{\sec\theta = -\sqrt{5}}$$

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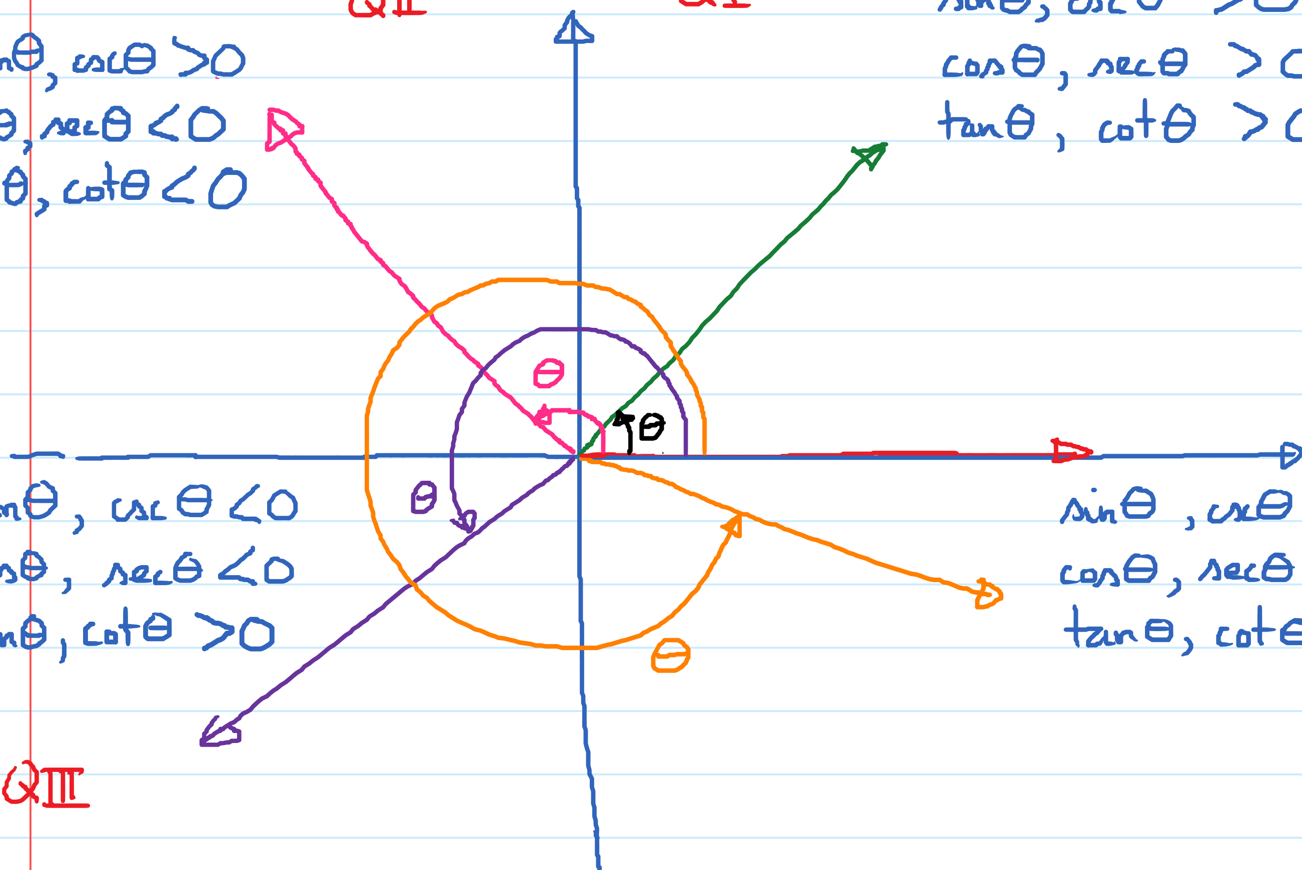
Determine the signs of trig functions of non-quadrantal angle



QI:
$\sin\theta, \csc\theta > 0$
$\cos\theta, \sec\theta > 0$
$\tan\theta, \cot\theta > 0$

QII:
$\sin\theta, \csc\theta > 0$
$\cos\theta, \sec\theta < 0$
$\tan\theta, \cot\theta < 0$

QIII:
$\sin\theta, \csc\theta < 0$
$\cos\theta, \sec\theta < 0$
$\tan\theta, \cot\theta > 0$

QIV:
$\sin\theta, \csc\theta < 0$
$\cos\theta, \sec\theta > 0$
$\tan\theta, \cot\theta < 0$

E.g. $\theta = -115^\circ$

$\sin(-115^\circ) < 0$, $\cos(-115^\circ) < 0$

$\tan(-115^\circ) > 0$

E.g. $\theta = 855^\circ$

$\sin(855^\circ) > 0$, $\cos(855^\circ)$, $\tan(855^\circ) < 0$

E.g. $\cos\theta < 0$, $\sin\theta < 0$

Which quadrant does the angle belong to? Q III

E.g. $\cot\theta < 0$, $\sec\theta < 0 \rightarrow \theta$ is in Q II.

E.g. $\cos\theta > 0$, $\sec\theta > 0 \rightarrow \theta$ is in Q I or Q IV.

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cot^2\theta = \csc^2\theta - 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad ; \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

E.g. Given: $\cos\theta = \frac{4}{5}$ and θ is in Q IV

Find $\sin\theta$

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{4}{5}\right)^2 = \frac{1\cdot 25}{1\cdot 25} - \frac{16}{25}$

$\sin^2\theta = \frac{9}{25} \longrightarrow \sin\theta = \pm\frac{3}{5}$

Since θ is in Q IV, $\sin\theta < 0$.

So, $\boxed{\sin\theta = -\frac{3}{5}}$

E.g. Given: $\sin\theta = \frac{1}{2}$ and θ is in Q II.

Find $\tan\theta$.

Sol: * Find $\cos\theta$ from Pythagorean identities:

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4}$

$\cos^2\theta = \frac{3}{4} \longrightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

Since θ is in Q II, $\cos\theta = -\frac{\sqrt{3}}{2}$

* Find $\tan\theta$ from Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{\cancel{2}}{\cancel{2}\sqrt{3}} = -\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$= \boxed{-\frac{\sqrt{3}}{3}}$

E.g. $\cos\theta = -\dfrac{\sqrt{3}}{2}$; $\tan\theta > 0$.

Find the remaining trig function values of θ.

Sol:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{\sqrt{3}}{2}\right)^2 = 1 - \frac{3}{4}$$

$$\sin^2\theta = \frac{1}{4} \longrightarrow \sin\theta = \pm\frac{1}{2}$$

θ is in Q III $\longrightarrow$ $\boxed{\sin\theta = -\dfrac{1}{2}}$.

$\boxed{\csc\theta = -2}$ (Reciprocal)

$\boxed{\sec\theta = -\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}}$ (Reciprocal)

$$\tan\theta \overset{\text{Quotient}}{=} \frac{\sin\theta}{\cos\theta} = \frac{\cancel{-}\frac{1}{2}}{\cancel{-}\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\boxed{\tan\theta = \dfrac{\sqrt{3}}{3}}$

$$\cot\theta = \frac{3}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$\boxed{\cot\theta = \sqrt{3}}$ $\longrightarrow$ Reciprocal.

Note: If we know the value of 1 trig function of θ and we know location of θ, then we can find the rest of them.

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The Range Values of Trig functions (the output values)

Function	Range
$\sin\theta$, $\cos\theta$	$[-1, 1]$
$\sec\theta$, $\csc\theta$	$(-\infty, -1] \cup [1, \infty)$
$\tan\theta$, $\cot\theta$	$(-\infty, \infty)$

E.g. $\cos\theta = -1.9$. $\longrightarrow$ Doesn't make sense.

$\csc\theta = -0.789 \longrightarrow$ Doesn't make sense.