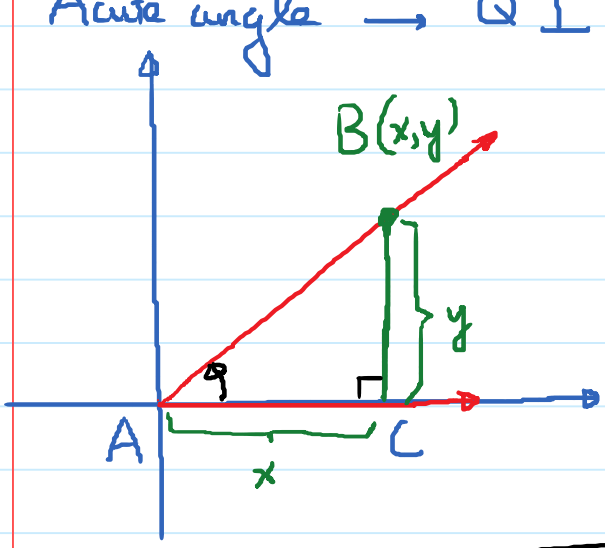
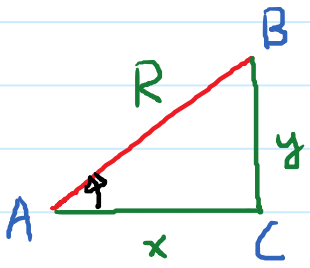
2.1. Trig. Functions of Acute Angles

Wednesday, January 30, 2019 10:21 AM

Acute angle → QI.





$$\sin A = \frac{y}{R} = \frac{\text{opp. to } A}{\text{hyp}} \; ; \quad \cos A = \frac{x}{R} = \frac{\text{adj. to } A}{\text{hyp}}$$

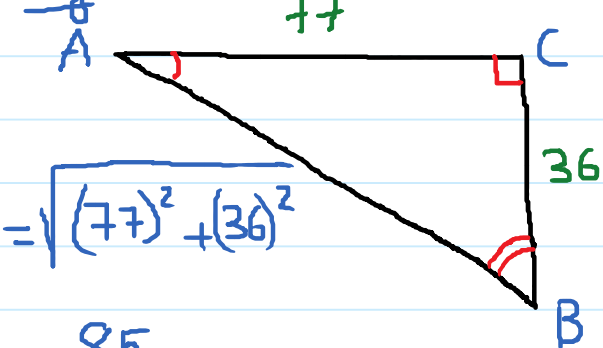
$$\tan A = \frac{y}{x} = \frac{\text{opp. to } A}{\text{adj. to } A} \; ; \quad \cot A = \frac{x}{y} = \frac{\text{adj.}}{\text{opp.}}$$

$$\sec A = \frac{R}{x} = \frac{\text{hyp}}{\text{adj.}} \; ; \quad \csc A = \frac{R}{y} = \frac{\text{hyp.}}{\text{opp.}}$$

Right-triangle based definition of trig functions.

SOH CAH TOA

E.g.



Find $\sin A$, $\cos A$, $\tan A$

$\sin B$, $\cos B$, $\tan B$.

$AB = \sqrt{(77)^2 + (36)^2}$

$= 85$

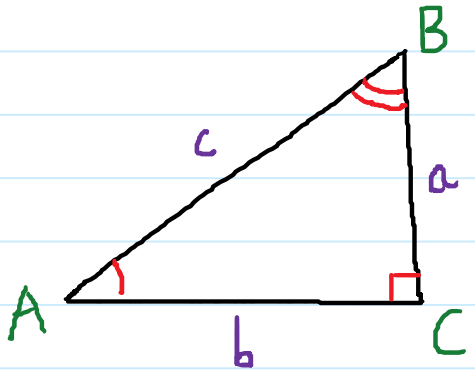
$$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{36}{85}$$

$\sin A = \frac{36}{85}$, $\cos A = \frac{77}{85}$, $\tan A = \frac{36}{77}$

$\sin B = \frac{77}{85}$; $\cos B = \frac{36}{85}$, $\tan B = \frac{77}{36}$

E.g. ΔABC is a right triangle with side lengths a, b, c.

$m\angle C = 90°$. Given: $a = 6\sqrt{2}$ and $c = 7\sqrt{2}$



Find $\csc B$, $\cot B$, $\sec A$, $\tan A$.

$a^2 + b^2 = c^2$

$(6\sqrt{2})^2 + b^2 = (7\sqrt{2})^2$

$\rightarrow b^2 = 98 - 72 = 26$

$\rightarrow b = \sqrt{26}$

$\csc B = \frac{hyp}{opp} = \frac{c}{b} = \frac{7\sqrt{2}}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{7\sqrt{52}}{26} = \frac{14\sqrt{13}}{26}$

$\csc B = \frac{7\sqrt{13}}{13}$

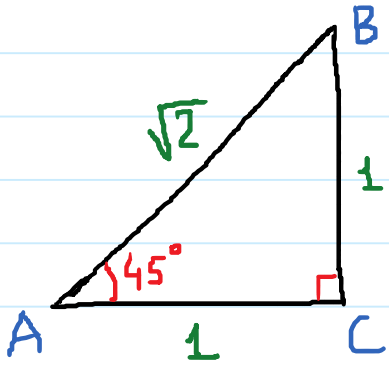
$\cot B = \frac{adj.}{opp.} = \frac{a}{b} = \frac{6\sqrt{2}}{\sqrt{26}} \rightarrow \ldots = \frac{6\sqrt{13}}{13}$

$\sec A = \frac{hyp.}{adj.} = \frac{c}{b} = \frac{7\sqrt{2}}{\sqrt{26}} = \frac{7\sqrt{13}}{13}$

$\tan A = \frac{6\sqrt{13}}{13}$

Trig Function Values of Special Triangles:

45° - 45° - 90° Triangle



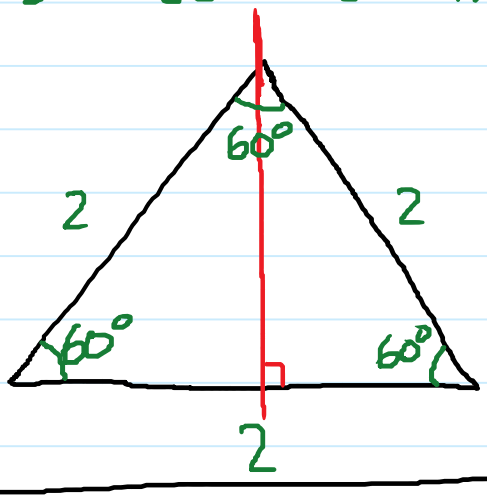
$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

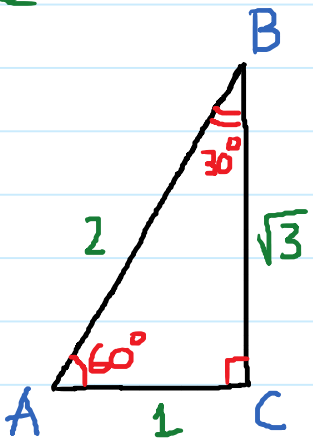
$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$; $\cot 45^\circ = 1$

$\sec 45^\circ = \sqrt{2}$; $\csc 45^\circ = \sqrt{2}$.

30° - 60° - 90° Triangle





$\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\cos 60^\circ = \frac{1}{2}$; $\tan 60^\circ = \sqrt{3}$

$\csc 60^\circ = \frac{2\sqrt{3}}{3}$; $\sec 60^\circ = 2$; $\cot 60^\circ = \frac{\sqrt{3}}{3}$

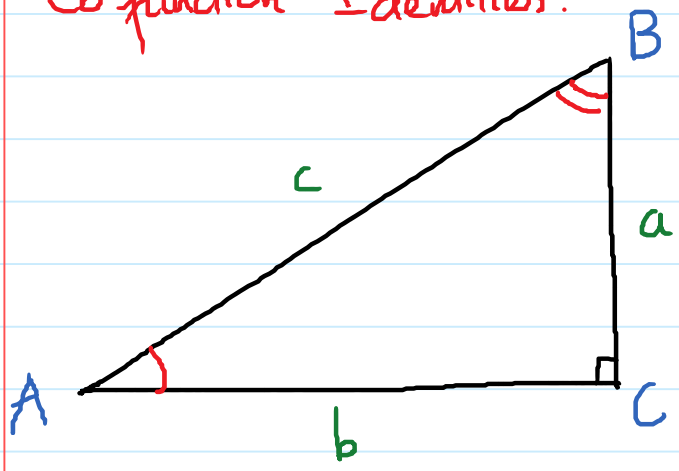
$\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\tan 30^\circ = \frac{\sqrt{3}}{3}$

$\csc 30^\circ = 2$; $\sec 30^\circ = \frac{2\sqrt{3}}{3}$; $\cot 30^\circ = \sqrt{3}$

Wednesday, January 30, 2019 11:13 AM

Cofunction Identities.

Note: $\angle A$ and $\angle B$ are complementary angles. ($\angle A + \angle B = 90$)



$\sin A = \frac{a}{c}$

$\cos B = \frac{a}{c}$

$\rightarrow \sin A = \cos B.$

Bottom line: If 2 angles are complementary, then sine of one is equal to cosine of the other, secant of one is equal to cosecant of the other, tangent of one is equal to cotangent of the other, etc

Cofunction: sine vs. cosine
secant vs. cosecant
tangent vs. cotangent.

$$\sin A = \cos(90^\circ - A); \quad \cos A = \sin(90^\circ - A)$$

$$\sec A = \csc(90^\circ - A); \quad \csc A = \sec(90^\circ - A)$$

$$\tan A = \cot(90^\circ - A); \quad \cot A = \tan(90^\circ - A)$$

Cofunction Identities.

E.g. $\sin 15^\circ = \cos 75^\circ$

$\tan 83^\circ = \cot 7^\circ$

E.g. All angles are acute.

$$\csc \alpha = \sec(\alpha + 30^\circ)$$

Find α.

The angles α and $\alpha + 30^\circ$ are complementary.

So, $\alpha + \alpha + 30^\circ = 90^\circ$

$2\alpha + 30^\circ = 90^\circ \rightarrow 2\alpha = 60^\circ$

$\rightarrow \boxed{\alpha = 30^\circ}$

E.g. All angles are acute.

$$\tan(3B - 31^\circ) = \cot(4B - 5^\circ)$$

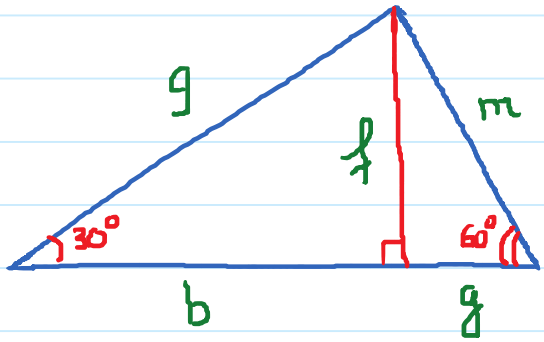
Find B.

$3B - 31^\circ + 4B - 5^\circ = 90^\circ$

$7B - 36^\circ = 90^\circ \rightarrow 7B = 126^\circ$

$\rightarrow \boxed{B = 18^\circ}$

E.g.



Find b.

$$\cos 30^\circ = \frac{b}{g} \longrightarrow b = g\cos 30^\circ$$

$$b = g \cdot \frac{\sqrt{3}}{2} = \frac{g\sqrt{3}}{2}$$

Find f.

$$\sin 30^\circ = \frac{f}{g} \longrightarrow f = g\sin 30^\circ$$

$$f = g \cdot \frac{1}{2} = \frac{g}{2}$$

Find m.

$$\sin 60^\circ = \frac{f}{m} = \frac{\frac{g}{2}}{m}$$

$$m = \frac{\frac{g}{2}}{\sin 60^\circ} = \frac{\frac{g}{2}}{\frac{\sqrt{3}}{2}} = \frac{g}{2} \cdot \frac{2}{\sqrt{3}} = \frac{18}{2\sqrt{3}}$$

$$= \frac{g}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 3\sqrt{3}$$