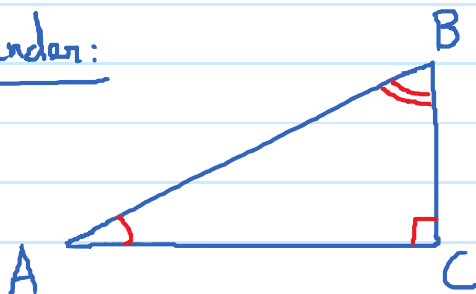


2.2. Trig Functions of Non-Acute Angles

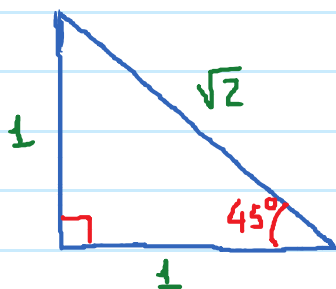
Monday, February 4, 2019

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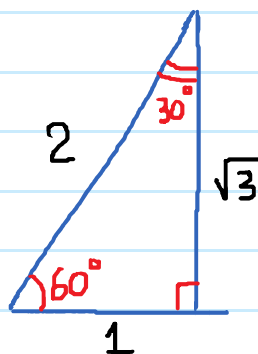
Reminder:



SOHCAHTOA



$45^\circ - 45^\circ - 90^\circ$

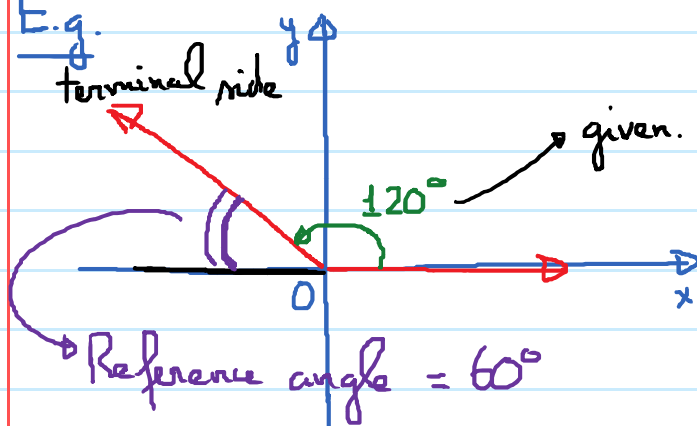


$60^\circ - 30^\circ - 90^\circ$

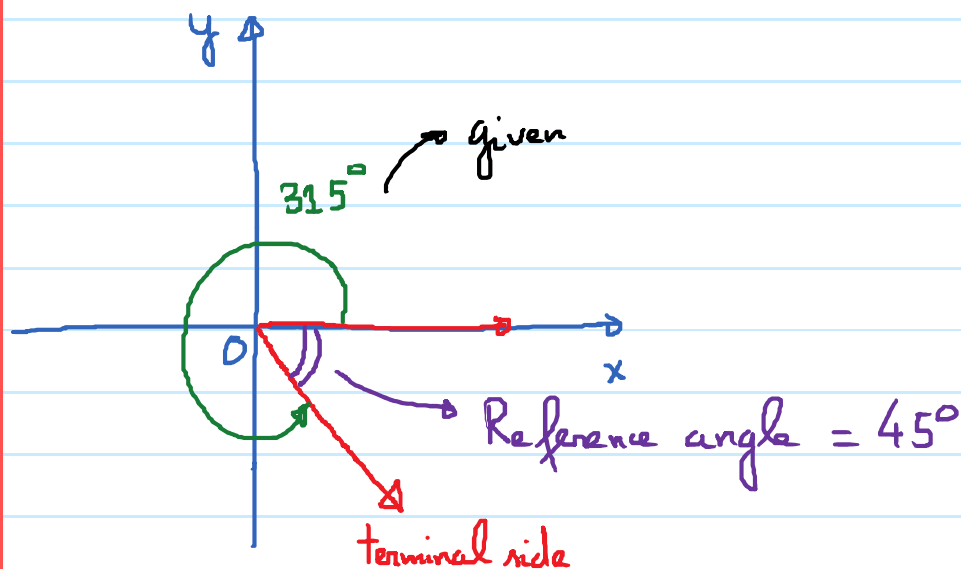
Reference Angle

Def: Given an angle, the reference angle for that angle is the positive acute angle made by the terminal side of the given angle and the x-axis.

E.g.



Q: Find the reference angle for $\theta = 120^\circ$



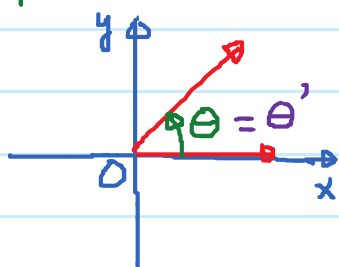
Process for finding the reference angle of a given angle θ

Step 1: If θ is not in $[0^\circ, 360^\circ]$, we find the coterminal angle of θ that is in $[0^\circ, 360^\circ]$ (by adding or subtracting appropriate multiple of 360° to θ)

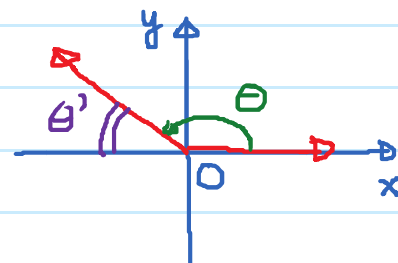
Reference angle

Step 2: Once θ is in $[0^\circ, 360^\circ]$, there are 4 cases:

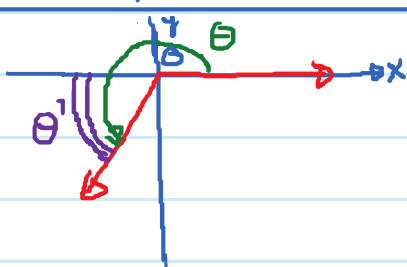
$$\theta' = \theta$$



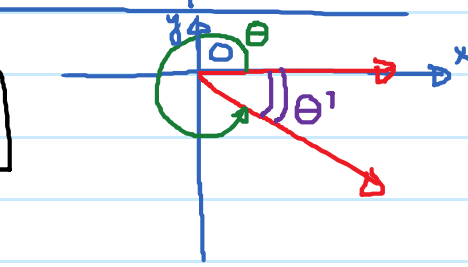
$$\theta' = 180^\circ - \theta$$



$$\theta' = \theta - 180^\circ$$



$$\theta' = 360^\circ - \theta$$



E.g. Find the reference angle for the given angle.

(a) -290° .

Step 1: Coterminal in $[0^\circ, 360^\circ]$: $\theta = -290^\circ + 360^\circ = 70^\circ$

Step 2: θ is in Q1. $\theta' = \theta = 70^\circ$

(b) 1130° ($1130^\circ / 360^\circ \approx 3.14 \rightarrow 3 \text{ rotations}$)

Step 1: $\theta = 1130^\circ - 3 \cdot 360^\circ = 50^\circ$

Step 2: θ is in Q1. $\theta' = \theta = 50^\circ$

Ex: (c) 927°

(d) -3428°

(c) Step 1: $927^\circ - 2 \cdot 360^\circ = 207^\circ$

Step 2: θ is in Q3. $\theta' = 207^\circ - 180^\circ = 27^\circ$

(d) Step 1: $\theta = -3428^\circ + 10 \cdot 360^\circ = 172^\circ$

Step 2: θ is in Q2: $\theta' = 180^\circ - 172^\circ = 8^\circ$

Finding trig function values for non quadrantal angles (that are related to 30° , 45° , 60° angles)

* Given an angle θ . (If θ is not in $[0^\circ, 360^\circ]$, we need to find the coterminal angle in $[0^\circ, 360^\circ]$ of θ)

Step 1: Find the reference angle θ' of θ .

(Note that θ' is acute)

Step 2: Use SOHCAHTOA on special right triangle to find the trig function values of θ' .

Step 3: Determine the correct sign of θ based on the quadrant of θ and adjust your answers in step 2 accordingly.

E.g. $\theta = 240^\circ$. Find the exact values of all the trig functions of θ .

Step 1: Reference angle θ' . $\theta' = 240^\circ - 180^\circ = 60^\circ$

Step 2: Find trig function values of θ'

$$\sin 60^\circ = \frac{\sqrt{3}}{2} ; \cos 60^\circ = \frac{1}{2} ; \tan 60^\circ = \sqrt{3}$$

$$\csc 60^\circ = \frac{2\sqrt{3}}{3} ; \sec 60^\circ = 2 ; \cot 60^\circ = \frac{\sqrt{3}}{3}$$

Step 3: Adjust the signs. 240° is in QIII

$$\sin 240^\circ = -\frac{\sqrt{3}}{2} ; \cos 240^\circ = -\frac{1}{2} ; \tan 240^\circ = \sqrt{3}$$

<0 <0 >0

$$\csc 240^\circ = -\frac{2\sqrt{3}}{3} ; \sec 240^\circ = -2 ; \cot 240^\circ = \frac{\sqrt{3}}{3}$$

<0 <0 >0

Ex. Find the exact value of :

(a) $\sin(-150^\circ)$ (b) $\cot 1035^\circ$ (c) $\cos(-300^\circ)$

(d) $\sec(750^\circ)$

(a) $\theta = -150^\circ + 360^\circ = 210^\circ$

$$\theta' = 210^\circ - 180^\circ = 30^\circ$$

$$\sin 30^\circ = \frac{1}{2} ; \sin \underbrace{(-150^\circ)}_{\substack{\text{QIII}}} = \boxed{-\frac{1}{2}}$$

① $\cot(1035^\circ)$; $1035^\circ / 360^\circ \approx 2 \dots$ rotations

$$\theta = 1035^\circ - 2 \cdot 360^\circ = 315^\circ \quad (\text{QIV})$$

$$\theta' = 360^\circ - 315^\circ = 45^\circ$$

$$\cot(45^\circ) = 1 ; \quad \underbrace{\cot(1035^\circ)}_{< 0} = \boxed{-1}$$

② $\cos(-300^\circ)$

$$\theta = -300^\circ + 360^\circ = 60^\circ \quad (\text{QI})$$

$$\theta' = 60^\circ \rightarrow \cos 60^\circ = \frac{1}{2}$$

$$\text{So, } \underbrace{\cos(-300^\circ)}_{> 0} = \boxed{\frac{1}{2}}$$

③ $\sec(750^\circ)$ $(750^\circ / 360^\circ \approx 2 \dots \text{ rotations})$

$$\theta = 750^\circ - 2 \cdot 360^\circ = 30^\circ \quad (\text{QI})$$

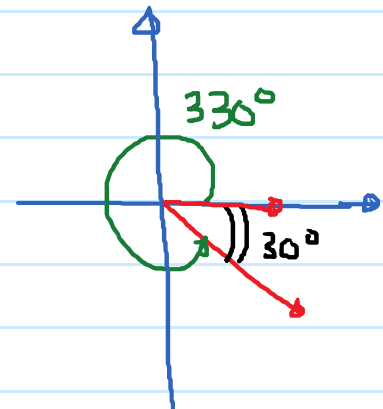
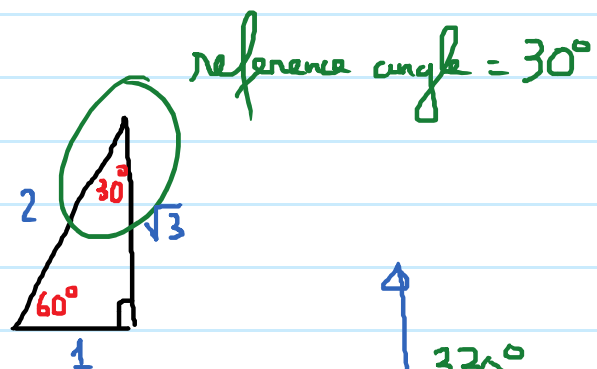
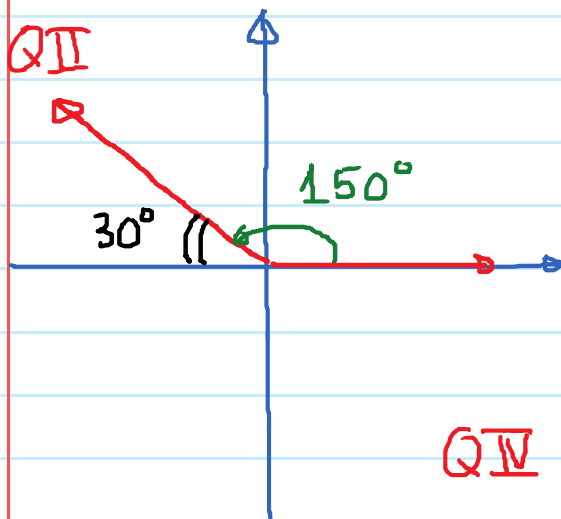
$$\theta' = 30^\circ ; \quad \sec(30^\circ) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec(750^\circ) = \boxed{\frac{2\sqrt{3}}{3}}$$

Find angle(s) given value(s) of trig functions.

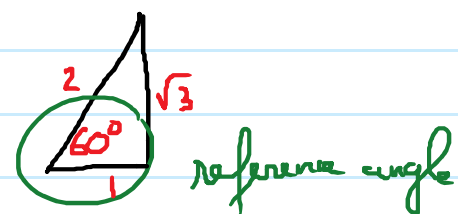
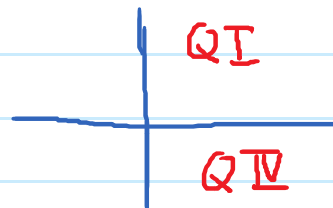
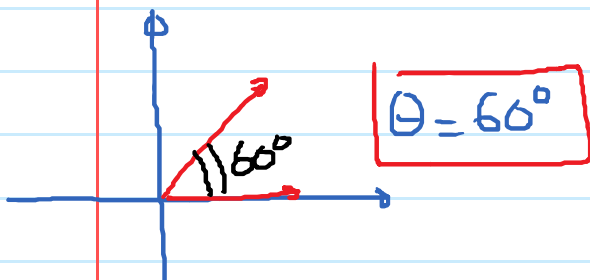
Find all values of θ in $[0^\circ, 360^\circ]$ and hence the given function value

(a) $\cot \theta = -\sqrt{3}$



Answer: $\theta = 150^\circ$ or $\theta = 330^\circ$

(b) $\csc \theta = \frac{2\sqrt{3}}{3}$



$\theta = 300^\circ$