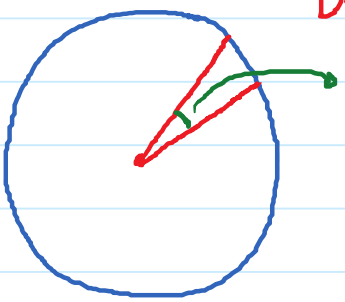
3.1. Radian Measures

Wednesday, February 13, 2019 10:27 AM

1°

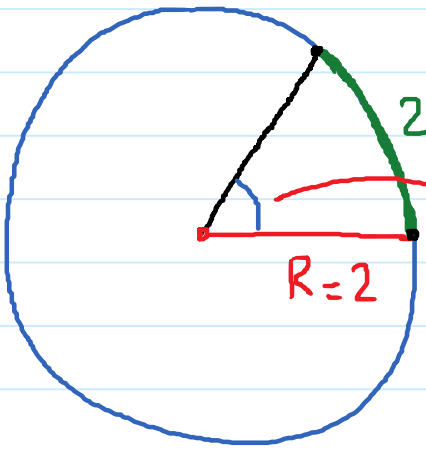


Divide the circle into 360 equal parts

central angle = 1°

1 radian

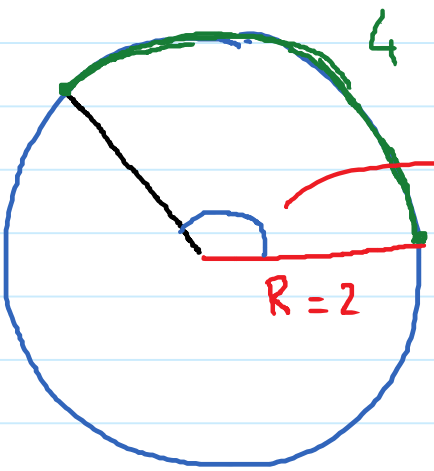
Circle with radius = 2 inches



measure an arc with length = R

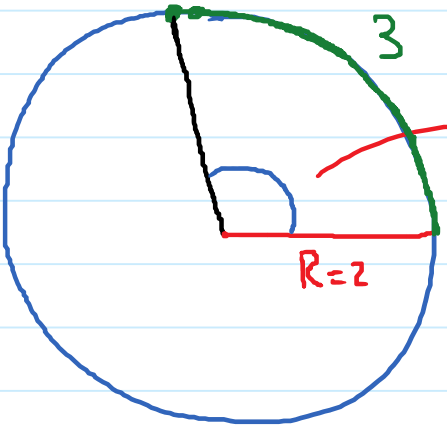
measure of this central angle

is exactly 1 radian.



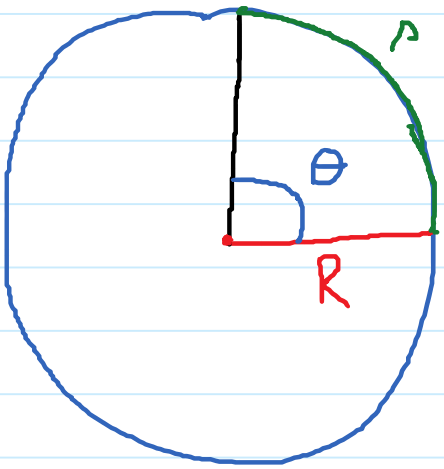
measure of this central angle

is 2 radians



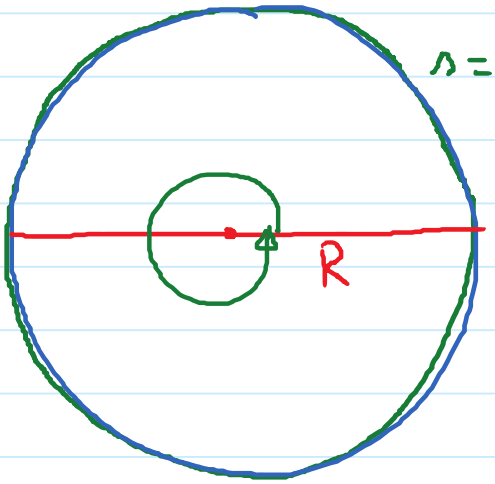
measure of this central angle

is $\frac{3}{2} = 1.5$ radians.



Radius = R ; arc length = s

In radian, what is the measure of the central angle θ in radians is $\boxed{\theta = \frac{s}{R}}$



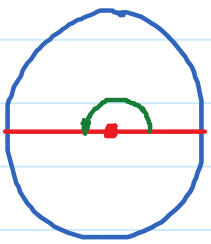
$s = 2\pi R$

So, $\theta = \frac{s}{R} = \frac{2\pi \cancel{R}}{\cancel{R}} = 2\pi$

The central angle which is a complete rotation has measure 2π (radians)

So, $360^\circ = 2\pi$ radians

$\longrightarrow 180^\circ = \pi$ radians



$\longrightarrow \boxed{1^\circ = \frac{\pi}{180} \text{ radians}}$

$\boxed{1 \text{ radian} = \frac{180}{\pi} \text{ degrees}}$

To convert from

degree $\xrightarrow{\text{multiply by } \frac{\pi}{180}}$ radian

radian $\xrightarrow{\text{multiply by } \frac{180}{\pi}}$ degree

Ex. Convert the following angles to radians

(a) 30° $\frac{\pi}{6}$

(b) 45° $\frac{\pi}{4}$

(c) 60° $\frac{\pi}{3}$

(d) 90° $\frac{\pi}{2}$

(e) 120° $\frac{2\pi}{3}$

(f) 135° $\frac{3\pi}{4}$

Ex. Convert the following angles to degrees

(a) $\frac{5\pi}{6} \cdot \frac{180}{\pi}$ 150°

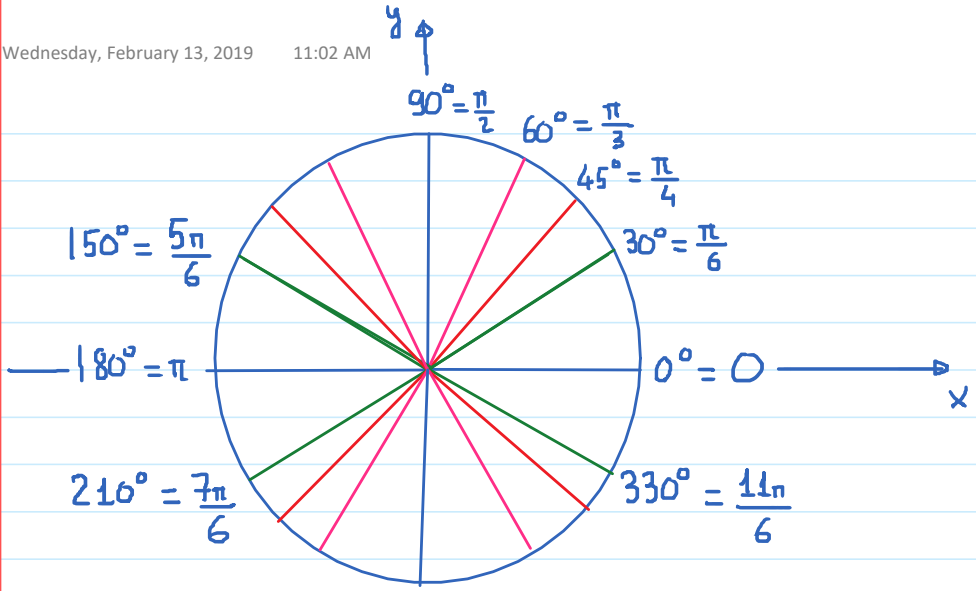
(b) $\frac{7\pi}{6}$ 210°

(c) $\frac{11\pi}{6}$ 330°

(d) $\frac{3\pi}{4}$ 135°

(e) $\frac{5\pi}{4}$ 225°

(f) $\frac{7\pi}{4}$ 315°



HW: Fill out the rest of circle.

Important family of angles that have same reference angle.

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ Reference angle: $\frac{\pi}{6}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ Reference angle: $\frac{\pi}{4}$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ Reference angle: $\frac{\pi}{3}$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6} = 30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4} = 45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3} = 60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

E.g. Find the exact value:

(a) $\tan\frac{2\pi}{3}$

Reference angle: $\frac{\pi}{3} \rightarrow \tan\frac{\pi}{3} = \sqrt{3}$

Quadrant: II $\rightarrow \tan\frac{2\pi}{3} = -\sqrt{3}$

(b) $\sec\frac{7\pi}{4}$

Reference angle: $\frac{\pi}{4} \rightarrow \sec\frac{\pi}{4} = \sqrt{2}$

Quadrant: IV $\rightarrow \sec\frac{7\pi}{4} = \sqrt{2}$

(c) $\sin\left(-\frac{7\pi}{6}\right)$

Coterminal: $-\frac{7\pi}{6} + 2\pi = \frac{5\pi}{6}$

Reference: $\frac{\pi}{6} \rightarrow \sin\frac{\pi}{6} = \frac{1}{2}$

Quadrant: $\sin\left(-\frac{7\pi}{6}\right) = \frac{1}{2}$

(d) $\cos\left(-\frac{14\pi}{3}\right)$

coterminal: $-\frac{14\pi}{3} + 3\cdot 2\pi = -\frac{14\pi}{3} + 6\pi = \frac{4\pi}{3}$

reference: $\frac{\pi}{3} \rightarrow \cos\frac{\pi}{3} = \frac{1}{2}$

Quadrant : III ; $\cos\left(-\frac{14\pi}{3}\right) = -\frac{1}{2}$.

(e) $\cos(3\pi) = -1$

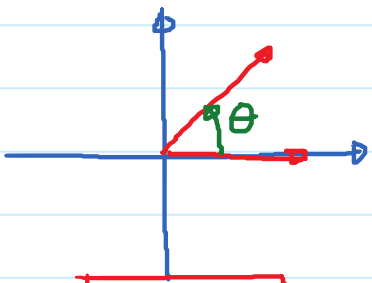
$\cos(2019\pi) = -1$

$\cos(2020\pi) = 1$

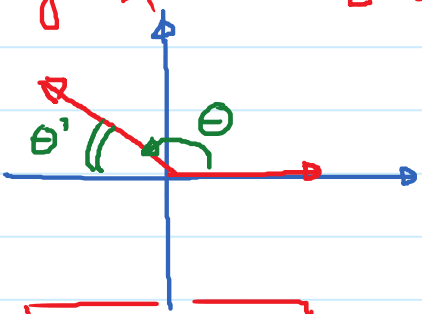
$\cos(k\pi)$ < -1 if k is odd; 1 if k is even

$\sin(k\pi) = 0$

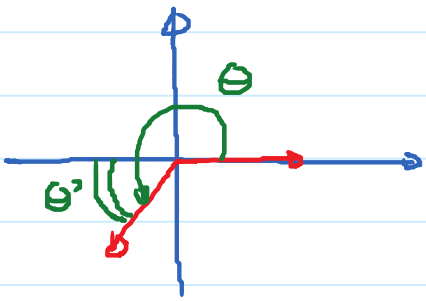
Formula for finding reference angle of θ in $[0, 2\pi]$



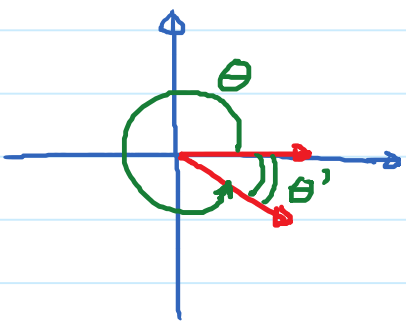
$\theta' = \theta$



$\theta' = \pi - \theta$



$\theta' = \theta - \pi$



$\theta' = 2\pi - \theta$