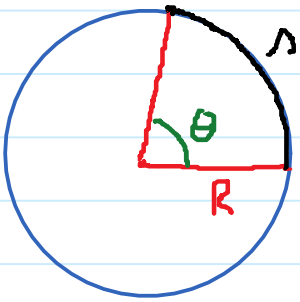
3.2. Applications of Radian Measures

Wednesday, February 13, 2019 11:38 AM

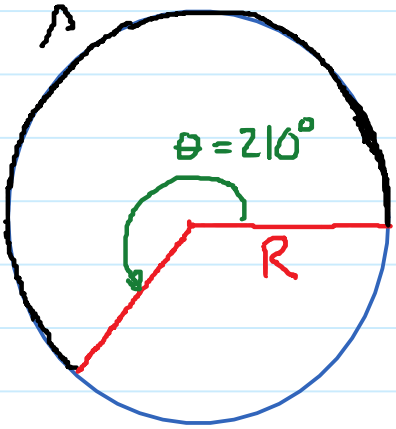
Arc Length

$$s = R \cdot \theta$$



Note: θ must be measured in radians

E.g. Find the length of the arc intercepted by a central angle $\theta = 210°$. Given $R = 25.6$ (cm)



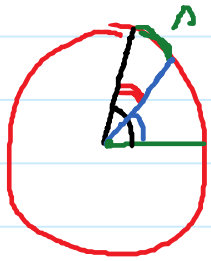
$$\theta = 210° \cdot \frac{\pi}{180} = \frac{7\pi}{6} \text{ (radians)}$$

$$s = R \cdot \theta = (25.6) \cdot \frac{7\pi}{6}$$

$$s \approx 93.8289 \text{ cm}$$

E.g. city 1: latitude = 42° N

city 2: latitude = 34° N



Radius of earth = 6400 km.

Distance between city 1 and city 2

$\theta = 42° - 34° = 8° \longrightarrow 8 \cdot \frac{\pi}{180}$ (radians)

Distance between 2 cities: $s = R \cdot \theta = 6400 \cdot \left(\frac{8\pi}{180}\right)$

$\simeq 893.61$ km

E.g. (2 gears problem)

Step 1: Distance that the smaller gear travels

$$s = R \cdot \theta = (2.5) \cdot \left(225 \cdot \frac{\pi}{180}\right) = \frac{25}{8}\pi.$$

Step 2: Find the angle that the larger gear rotates.

$$s = R \cdot \theta \longrightarrow \theta = \frac{s}{R} = \frac{\frac{25}{8}\pi}{4.8} = \frac{125}{192}\pi$$

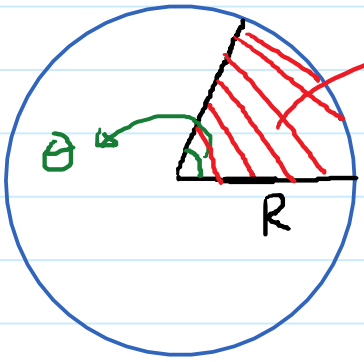
Convert to degrees: $\frac{125}{192}\pi \cdot \frac{180}{\pi} \simeq 117.2°$

E.g. (Pulley raising weight)

$s = 11.4$; $\theta = 51.6°$. Find R

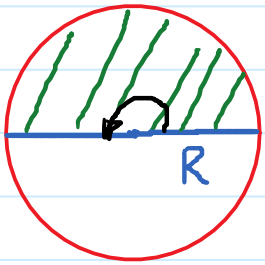
$$R = \frac{s}{\theta} = \frac{11.4}{(51.6) \cdot \frac{\pi}{180}} = 12.66 \text{ (m)}$$

Area of a sector of a circle



$A = ?$

central angle	Area
θ	?



central angle:	Area:
π	$\frac{\pi R^2}{2}$
1	$\frac{R^2}{2}$

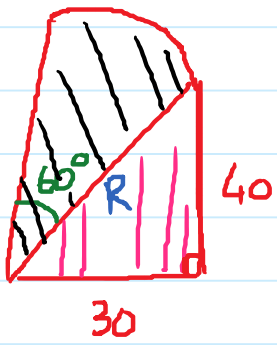
$\pi \longrightarrow \frac{\pi R^2}{2}$

$\theta \longrightarrow ? \quad \frac{\theta R^2}{2}$

$$\boxed{A = \frac{1}{2} R^2 \theta}$$

(Note: θ needs to be in radians)

E.g



$\theta = 60^\circ \longrightarrow \Theta = \frac{\pi}{3}$

$R = \sqrt{(30)^2 + (40)^2} = 50$

Area of sector $= \frac{1}{2} \cdot (50)^2 \cdot \frac{\pi}{3}$

$= \frac{1250\pi}{3}$

Area of $\Delta = \frac{1}{2} \cdot (30) \cdot (40) = 600$.

Total area $= \frac{1250\pi}{3} + 600 = \ldots$

E.g. $A = \frac{1}{2} R^2 \theta \longrightarrow 4 = \frac{1}{2}(4)^2 \cdot \theta$

$\longrightarrow 4 = \frac{1}{2} \cdot 16 \cdot \theta = 8\theta \rightarrow \theta = \frac{4}{8} = \frac{1}{2}$.