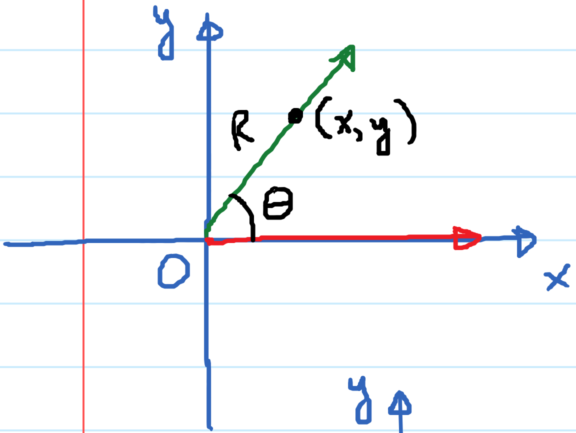
3.3. The Unit circle and circular functions

Monday, February 18, 2019 11:12 AM

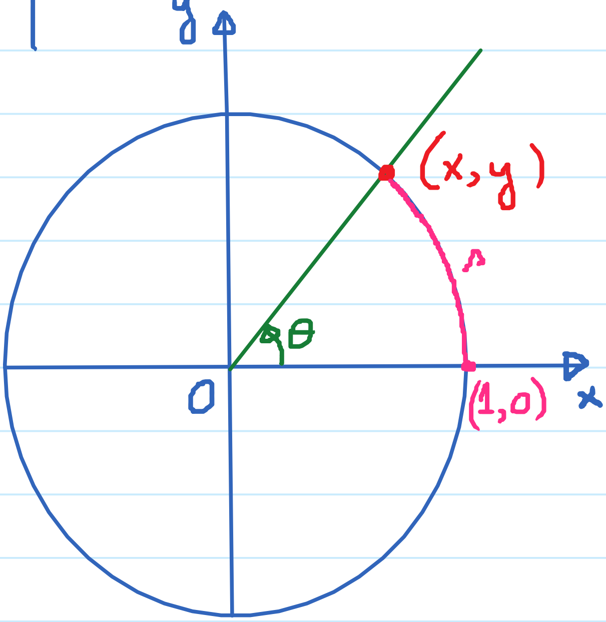
Recall:



$$\sin\theta = \frac{y}{R} \qquad \tan\theta = \frac{y}{x}$$

$$\cos\theta = \frac{x}{R}$$

Radius = R = 1 (Unit circle)



$$\sin\theta = y \ ; \ \tan\theta = \frac{y}{x}$$

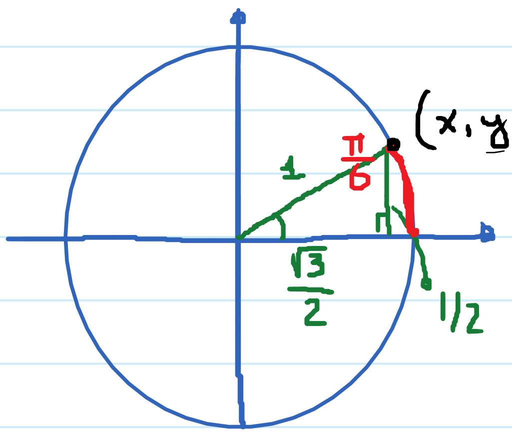
$$\cos\theta = x$$

$$s = \underbrace{R}_{1} \cdot \theta \longrightarrow s = \theta.$$

(radian)

$$\longrightarrow \sin(s) = y \ ; \ \cos(s) = x \ ; \ \tan(s) = \frac{y}{x}.$$

E.g. $s = \frac{\pi}{6}$



$$(x, y) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

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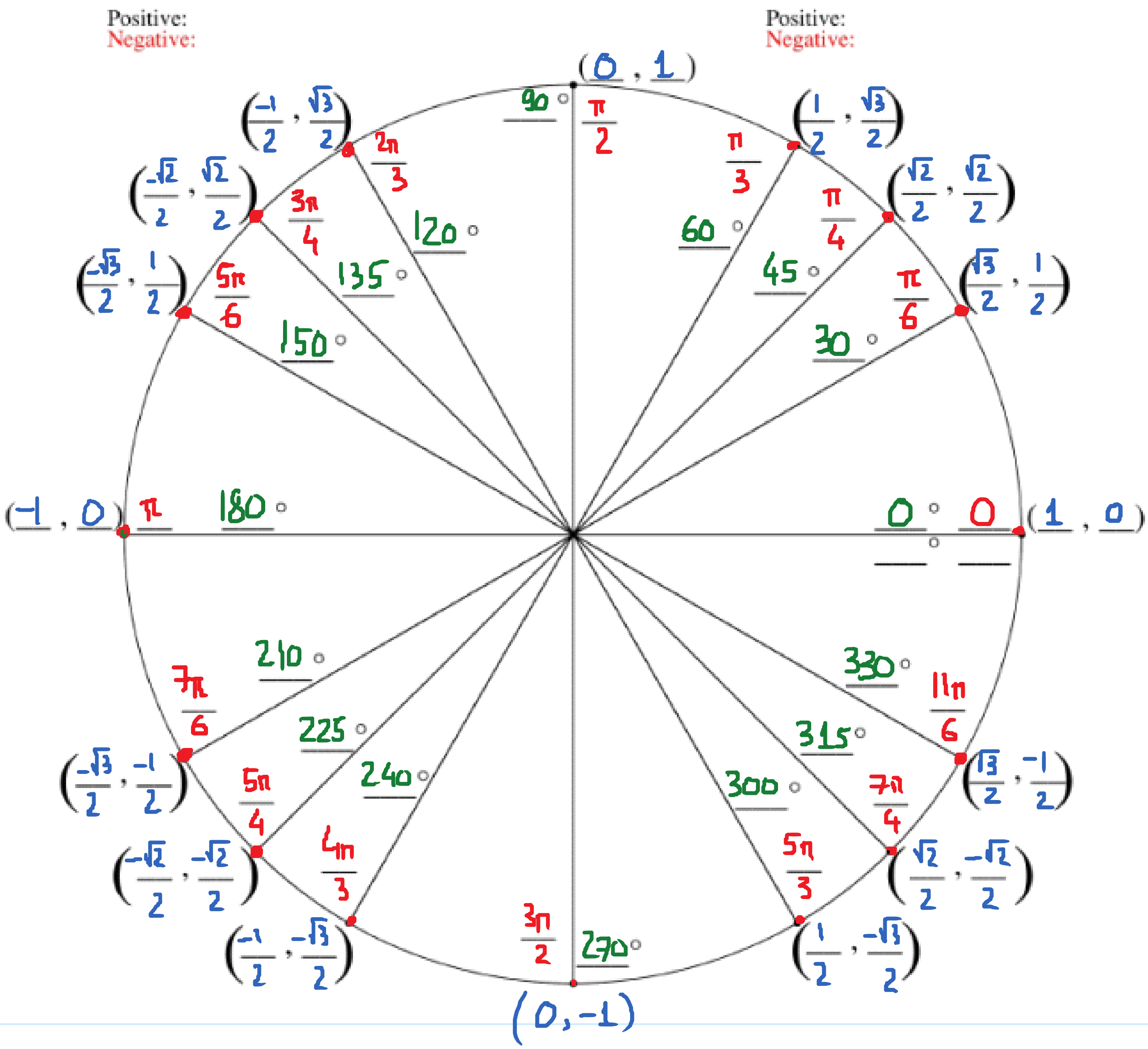


Find $\cos\frac{3\pi}{2}$? $\cos\frac{3\pi}{2} = 0$; $\cos\frac{17\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin\frac{11\pi}{6} = -\frac{1}{2}$; $\sin\frac{13\pi}{3} = \frac{\sqrt{3}}{2}$

1st use of the unit circle:

We can find sine and cosine of any angle related to one of the families:

I $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

II $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

III $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

IV $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

by identifying which point on the unit circle that angle corresponds to and use the x and y coordinates of the points

E.g. Find $\cos\left(\frac{37\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$$\frac{37\pi}{6} - \boxed{\frac{36\pi}{6}} = \frac{\pi}{6}$$

18π

$9 \cdot 2\pi$

E.g. Find $\sin\left(-\frac{29\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$$-\frac{29\pi}{3} + \frac{30\pi}{3} = \frac{\pi}{3}$$

2^{nd} use of unit circle:

Find s given $\cos s$ or $\sin s$ or the value of any trig function of s.

E.g. Find all the values of s in $[0, 2\pi)$ such that $\cos(s) = -\frac{1}{2}$.

Ans: $s = \frac{2\pi}{3}$ or $s = \frac{4\pi}{3}$.

E.g. Find all the values of s in $\left[\frac{3\pi}{2}, 2\pi\right)$ such that $\sin s = -\frac{\sqrt{2}}{2}$

Ans: $s = \frac{7\pi}{4}$.

E.g. Find all the values of s in $[0, 2\pi)$ such that $\tan s = \frac{\sqrt{3}}{3}$

Ans: $s = \frac{\pi}{6}$ or $s = \frac{7\pi}{6}$

E.g. Find all the values of s in $\left[\frac{\pi}{2}, \pi\right)$ such that $\tan(s) = -1$

Answer : $s = \frac{3\pi}{4}$

E.g. Find all the values of s such that $\sin(s) = \frac{1}{2}$

Answer : $\left.\begin{aligned} s &= \frac{\pi}{6} + k\cdot 2\pi \\ s &= \frac{5\pi}{6} + k\cdot 2\pi \end{aligned}\right\}$ where k is an integer, i.e, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$

Using calculators to find values of trig functions or to find s.

E.g. $\sin(0.6109) \simeq 0.5736\ldots$

Switch calculator into Rad mode.

E.g. Find s in $\left[0, \frac{\pi}{2}\right)$ such that $\tan(s) = 0.2126$.

$\underbrace{2^{nd} \longrightarrow \tan}_{\tan^{-1}} \longrightarrow 0.2126 \longrightarrow s = 0.20984$ (in rad if cal is in rad)

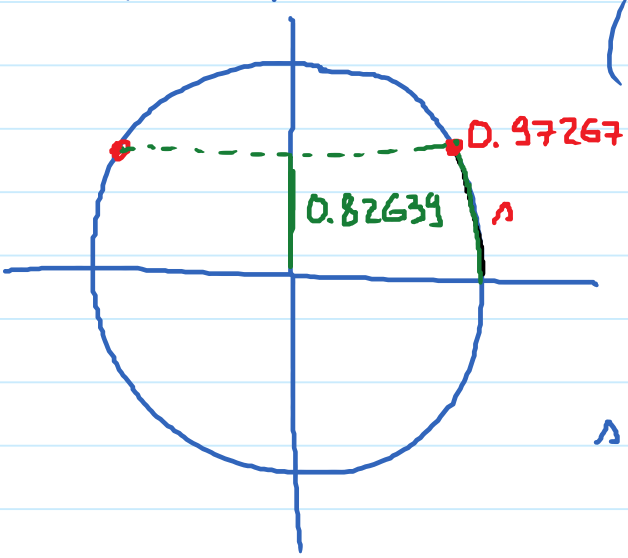
E.g. Find s in $[0, 2\pi)$ such that

$$\sin s = 0.82639.$$

$2^{nd} \to \sin \to 0.82639 \to \simeq \boxed{0.97267} < \frac{\pi}{2}$

$s = 0.97267$

$\left(\frac{\pi}{2} \simeq 1.571\right)$



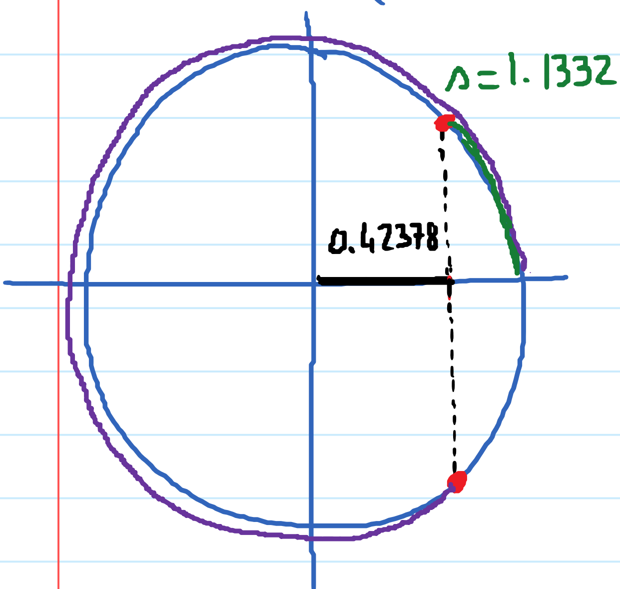
1^{st} solution: $\boxed{s = 0.97267}$

2^{nd} solution:

$s = \pi - 0.97267 = \boxed{2.1689}$

E.g. Find s in $[0, 2\pi)$ such that

$$\cos(s) = 0.42378.$$



$\cos^{-1}(0.42378) = 1.1332.$

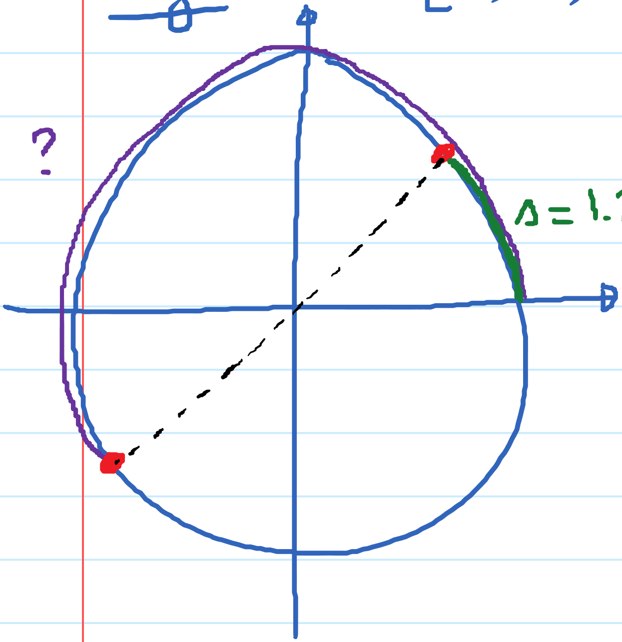
1^{st} solution: $\boxed{s = 1.1332}$

2^{nd} solution: $s = 2\pi - 1.1332$

$\boxed{s = 5.15}$

E.g. s in $[0, 2\pi)$. $\tan s = 2.75$

$\tan^{-1}(2.75) = 1.222$.



1^{st} sol: $s = \boxed{1.222}$

2^{nd} sol: $s = \pi + 1.222$

$s = \boxed{4.3636}$