4.1. Graphs of Sine and Cosine Functions

Monday, February 25, 2019 10:30 AM

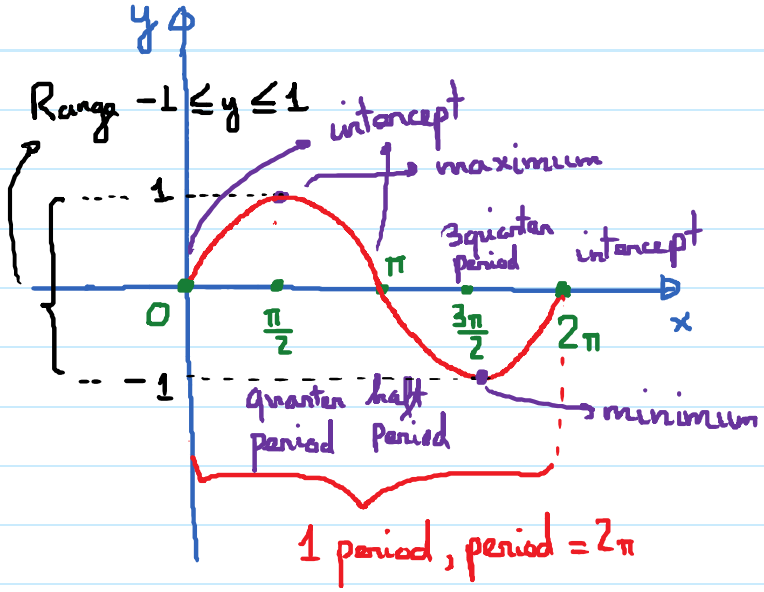
(1) Basic Sine and Cosine Curves.

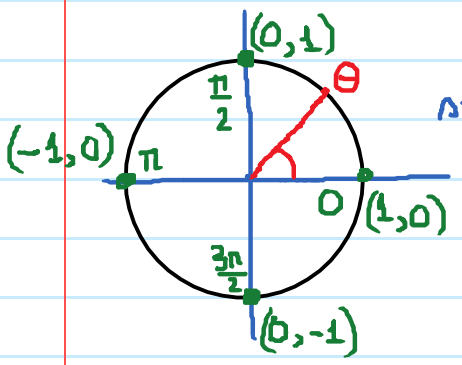
$y = \sin x$. To sketch the graph, we find 5 key points where x is in $[0, 2\pi]$

Key points

angle → x | sine of the angle → $y = \sin x$

x	$y = \sin x$	
0	0	$(0,0)$
$\frac{\pi}{2}$	1	$(\frac{\pi}{2}, 1)$
π	0	$(\pi, 0)$
$\frac{3\pi}{2}$	-1	$(\frac{3\pi}{2}, -1)$
2π	0	$(2\pi, 0)$





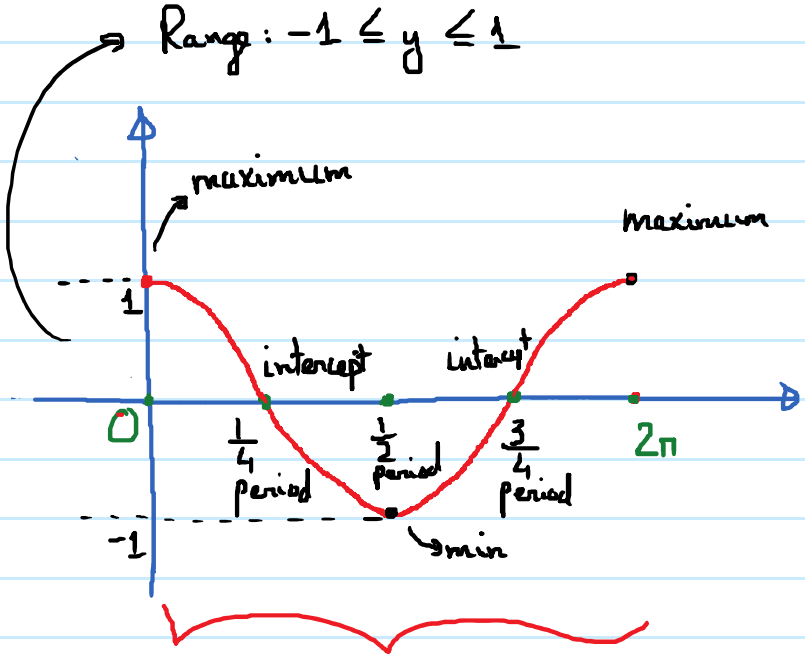
$\sin(\theta + 2\pi) = \sin\theta$

$y = \cos x$

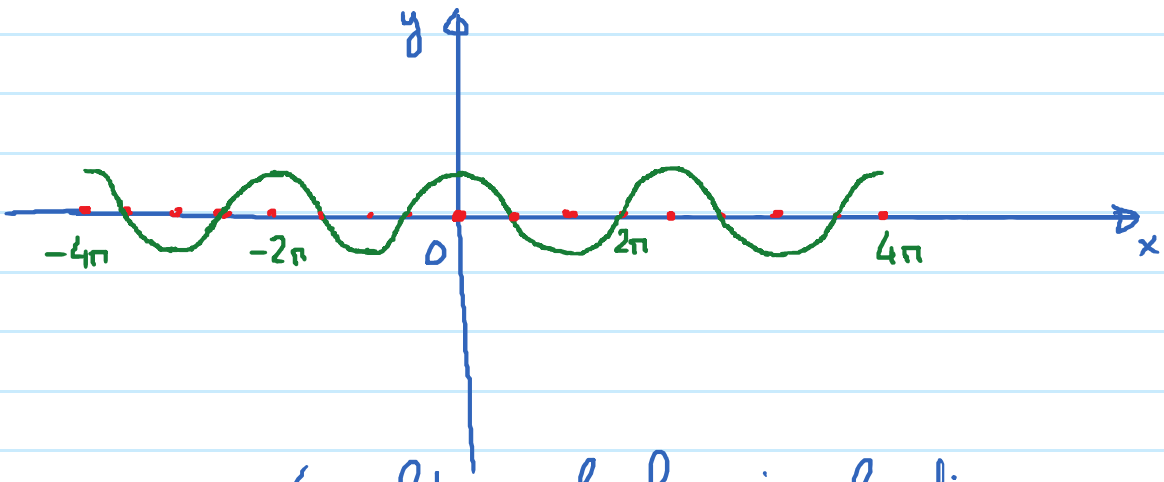
Key points

angle → x

x	$y = \cos x$	
0	1	$(0, 1)$
$\frac{\pi}{2}$	0	$(\frac{\pi}{2}, 0)$
π	-1	$(\pi, -1)$
$\frac{3\pi}{2}$	0	$(\frac{3\pi}{2}, 0)$
2π	1	$(2\pi, 1)$



1 period $= 2\pi$



Complete graph of cosine function

Summary: In 1 period $[0, 2\pi]$

Basic Sine Curve: Intercept Max Intercept Min Intercept

$(0,0)$ $(\frac{\pi}{2}, 1)$ $(\pi, 0)$ $(\frac{3\pi}{2}, -1)$ $(2\pi, 0)$

Basic Cosine curve: Max Intercept Min Intercept Max

$(0,1)$ $(\frac{\pi}{2}, 0)$ $(\pi, -1)$ $(\frac{3\pi}{2}, 0)$ $(2\pi, 1)$

(2) Graph functions of the form $y = a\sin x$ or

$y = a\cos x$

E.g. Sketch the graph of $y = 2\sin x$ on the interval

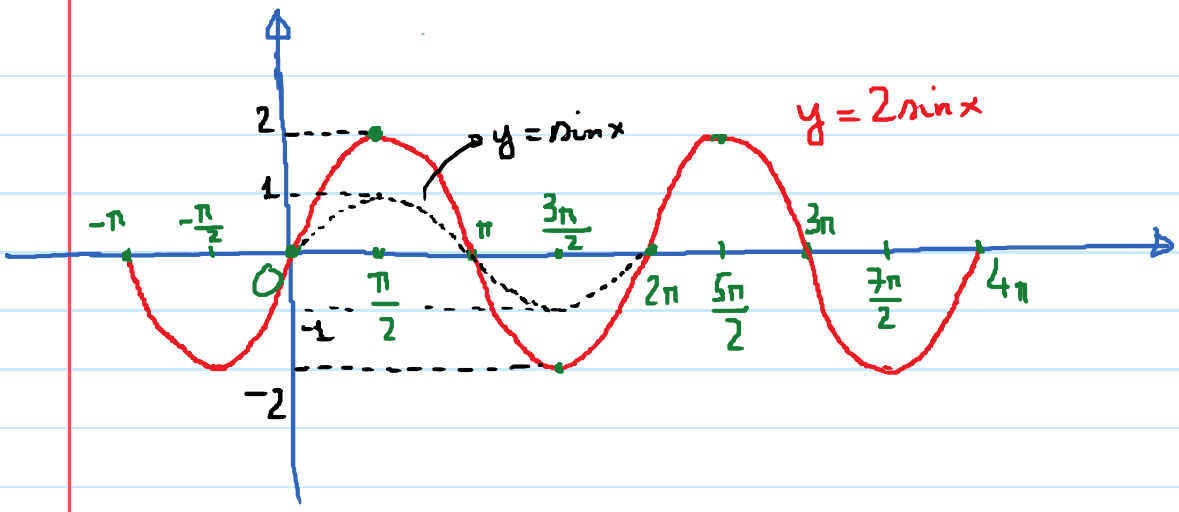
$[-\pi, 4\pi]$

$y = 2\sin x = 2(\sin x)$

→ the y-values of the key points will be multiplied by 2

Intercept Max Intercept Min Intercept

$(0,0)$ $(\frac{\pi}{2}, 2)$ $(\pi, 0)$ $(\frac{3\pi}{2}, -2)$ $(2\pi, 0)$



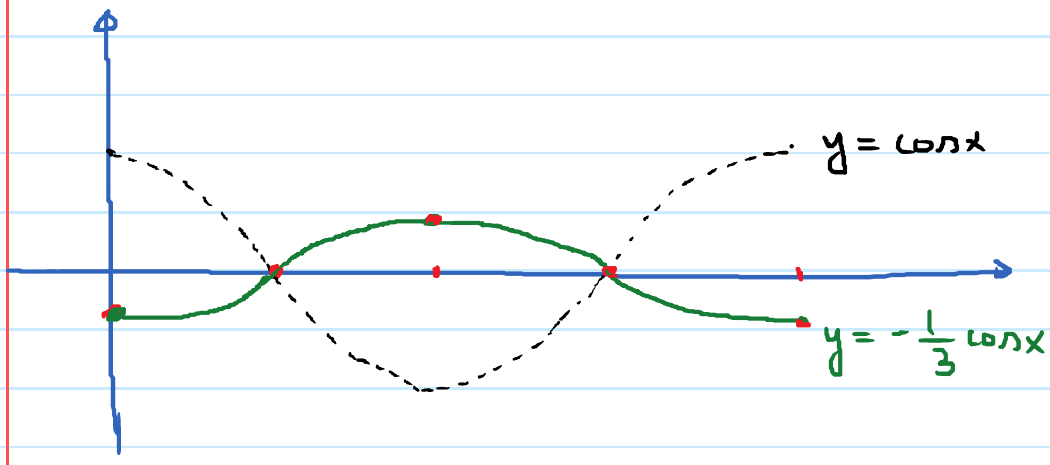
Def. of Amplitude of Sine and Cosine Function.

The **amplitude** of $y = a\sin x$ or $y = a\cos x$ is given by $\boxed{\text{Amplitude} = |a|}$

E.g. Graph $y = -\frac{1}{3}\cos x$ on $[0, 2\pi]$ Amplitude $= \frac{1}{3}$

$a = -\frac{1}{3}$

Min	Intercept	Max	Intercept	Min
$(0, -\frac{1}{3})$	$(\frac{\pi}{2}, 0)$	$(\pi, \frac{1}{3})$	$(\frac{3\pi}{2}, 0)$	$(2\pi, -\frac{1}{3})$



$$\text{Amplitude} = \frac{1}{2}(\max - \min)$$

(2) Graph of functions of the form $y = a\sin(bx)$ or $y = a\cos(bx)$ (Assume $b > 0$)

E.g. Sketch the graph of $y = \sin\left(\frac{1}{2}x\right)$ $\left(b = \frac{1}{2}\right)$ in 1 period.

	Intercept	Max	Intercept	Min	Intercept
Basic	$(0,0)$	$\left(\frac{\pi}{2},1\right)$	$(\pi,0)$	$\left(\frac{3\pi}{2},-1\right)$	$(2\pi,0)$

$y = \sin\left(\frac{1}{2}x\right)$ — $\frac{1}{2}x$ → angle

angle $= 0$

angle $= 2\pi$

} 1 period

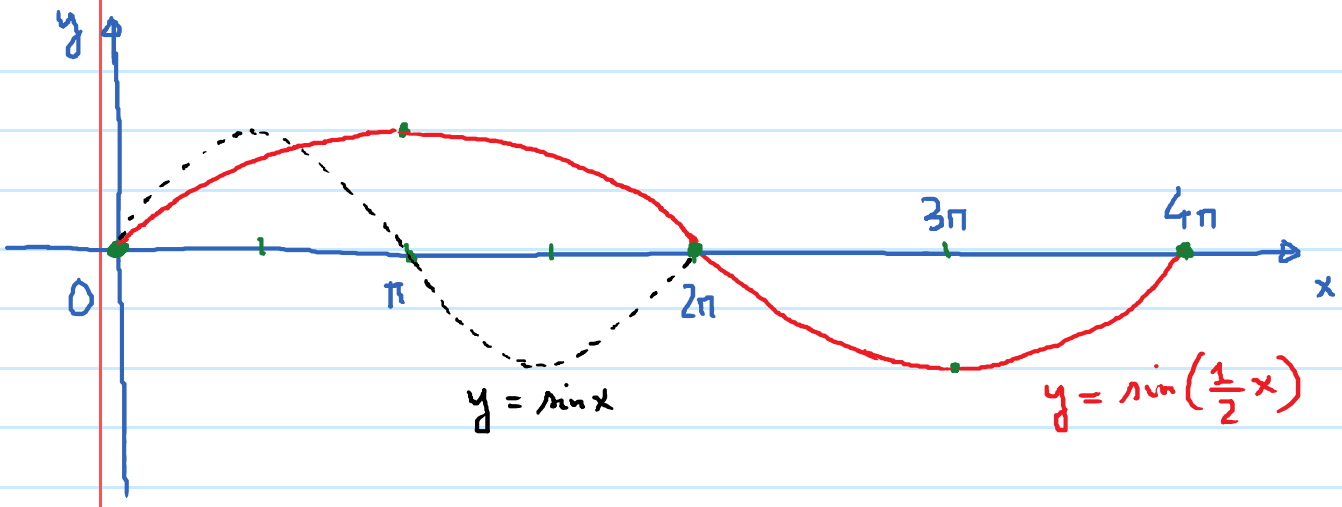
$\frac{1}{2}x = 0 \rightarrow x = 0$

$\frac{1}{2}x = 2\pi \rightarrow x = 4\pi$

} 1 period

$(0,0)$ $(\pi,1)$ $(2\pi,0)$ $(3\pi,-1)$ $(4\pi,0)$

Key points of $y = \sin\left(\frac{1}{2}x\right)$



Formula for period of $y = a\cos(bx)$ or $y = a\sin(bx)$

(assume $b > 0$)

$$\text{Period} = \frac{2\pi}{b}$$

(Reason: $y = a\sin(\boxed{bx})$ ← angle

angle $= 0 \rightarrow bx = 0 \rightarrow x = 0$

angle $= 2\pi \rightarrow bx = 2\pi \rightarrow x = \frac{2\pi}{b}$

1 period goes from 0 to $\frac{2\pi}{b}$.)

E.g. Find the amplitude and period and sketch the graph in 1 period for

$$y = -\frac{1}{2}\cos(\pi x) \; ; \quad a = -\tfrac{1}{2} \; ; \; b = \pi .$$

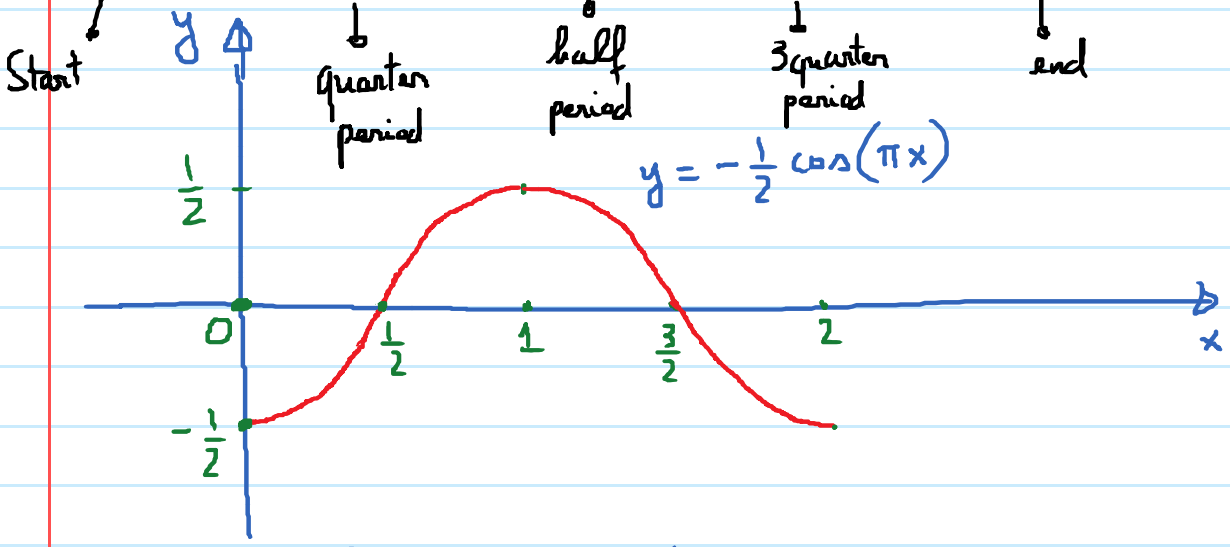
Amplitude : $|a| = \left|-\frac{1}{2}\right| = \frac{1}{2}$.

Period $= \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$

Min Intercept Max Intercept Min

$(0, -\frac{1}{2})$ $(\frac{1}{2}, 0)$ $(1, \frac{1}{2})$ $(\frac{3}{2}, 0)$ $(2, -\frac{1}{2})$

Start — Quarter period — half period — 3quarter period — end



E.g. Graph $y = 3\sin\left(\frac{\pi}{2}x\right) + 1$ in one period.

$a = 3 \rightarrow$ amplitude $= 3$

$b = \frac{\pi}{2} \rightarrow$ period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{2}} = 4$

"Intercept" Max "Intercept" Min "Intercept"

(0, 1) (1, 4) (2, 1) (3, -2) (4, 1)

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