4.2 Translations of Graphs of Sine and Cosine

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Goal: We will learn how to graph functions of the form.

$$y = a\sin(bx - c) + d$$

or

$$y = a\cos(bx - c) + d$$

Reminder:

Basic sine curve: $y = \sin x$

- Amplitude: 1
- Period: 2π
  - Start $= 0$
  - End $= 2\pi$

Pattern:	Intercept	Max	Intercept	Min	Intercept
	$(0,0)$	$(\frac{\pi}{2}, 1)$	$(\pi, 0)$	$(\frac{3\pi}{2}, -1)$	$(2\pi, 0)$
	Start	$\frac{1}{4}$ period	$\frac{1}{2}P$	$\frac{3}{4}P$	end

Basic cosine curve: $y = \cos x$ (Amplitude $= 1$, $p = 2\pi$)

Pattern:	Max	Intercept	Min	Intercept	Max
	$(0,1)$	$(\frac{\pi}{2}, 0)$	$(\pi, -1)$	$(\frac{3\pi}{2}, 0)$	$(2\pi, 1)$
	Start	$\frac{1}{4}P$	$\frac{1}{2}P$	$\frac{3}{4}P$	end

Variations: $y = a\sin(bx)$; $y = a\cos(bx)$

Amplitude: $|a|$; Period: $\dfrac{2\pi}{b}$ Start: 0 End: $\dfrac{2\pi}{b}$

Now, we consider: $y = a\sin(\boxed{bx-c})$ or $y = a\cos(\boxed{bx-c})$

angle angle

Amplitude: $|a|$;

When the angle $bx - c$ goes from 0 to 2π, the sine and cosine curves will go through 1 period.

To determine the interval for 1 period, we set $bx - c = 0$ and $bx - c = 2\pi$ and solve x.

$$x = \frac{c}{b} \qquad\qquad x = \frac{c+2\pi}{b} = \frac{c}{b} + \frac{2\pi}{b}$$

Interval for one period: $\left[\dfrac{c}{b}, \underbrace{\dfrac{c}{b} + \dfrac{2\pi}{b}}_{\text{End}}\right]$

Start End

$\left(\text{Note: } b > 0\right)$

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The graph of $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ have the following characteristics:

$\boxed{\text{Amplitude} = |a|}$; $\boxed{\text{Period} = \frac{2\pi}{b}}$

Start: $\frac{c}{b}$

End: $\frac{c}{b} + \frac{2\pi}{b}$.

The left and right endpoints of 1 period can be determined by setting $bx - c = 0$ and $bx - c = 2\pi$ and solve for x.

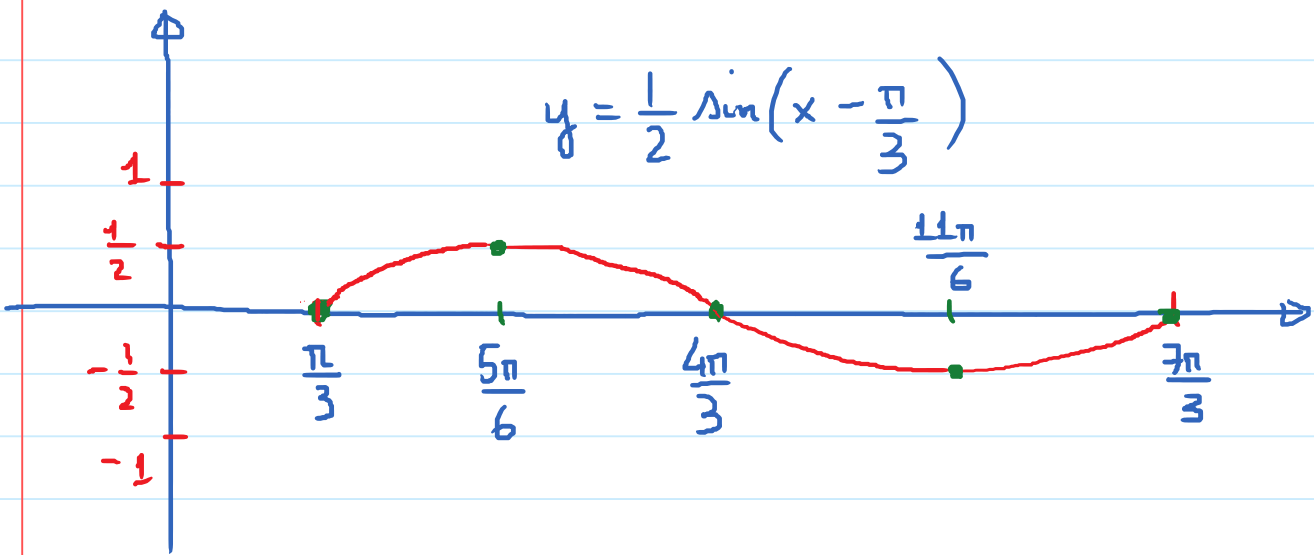
E.g. Sketch the graph of $y = \underbrace{\frac{1}{2}}_{a}\sin\left(x - \frac{\pi}{3}\right)$ in 1 period. $b = 1$

Amplitude $= \frac{1}{2}$, Period $= 2\pi$

Endpoints of 1 period: $x - \frac{\pi}{3} = 0$, $x - \frac{\pi}{3} = 2\pi$

$x = \frac{\pi}{3}$ (Start) ; $x = \frac{7\pi}{3}$ (End)

Intercept	Max	Intercept	Min	Intercept
$\left(\frac{\pi}{3}, 0\right)$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$	$\left(\frac{4\pi}{3}, 0\right)$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$	$\left(\frac{7\pi}{3}, 0\right)$



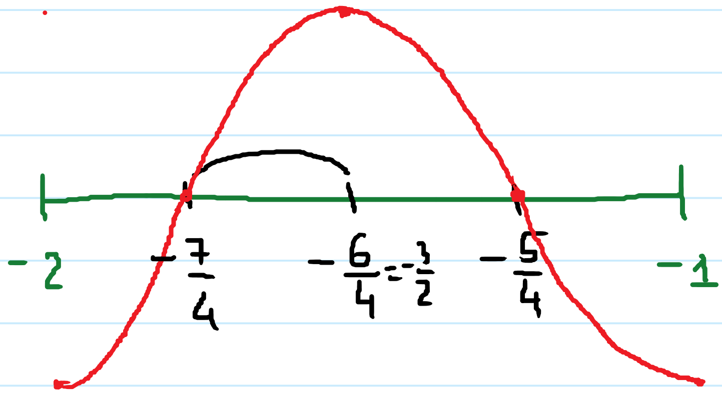
E.g. Find amplitude, period, 5 key points and sketch the graph of $y = -3\cos(2\pi x + 4\pi)$ in 1 period.

Amplitude $= 3$; Period $= \frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$.

Endpoints: $2\pi x + 4\pi = 0$; $2\pi x + 4\pi = 2\pi$

$x = -2$; $x = -1$.

Min	Intercept	Max	Intercept	Min
$(-2, -3)$	$\left(-\frac{7}{4}, 0\right)$	$\left(-\frac{3}{2}, 3\right)$	$\left(-\frac{5}{4}, 0\right)$	$(-1, -3)$



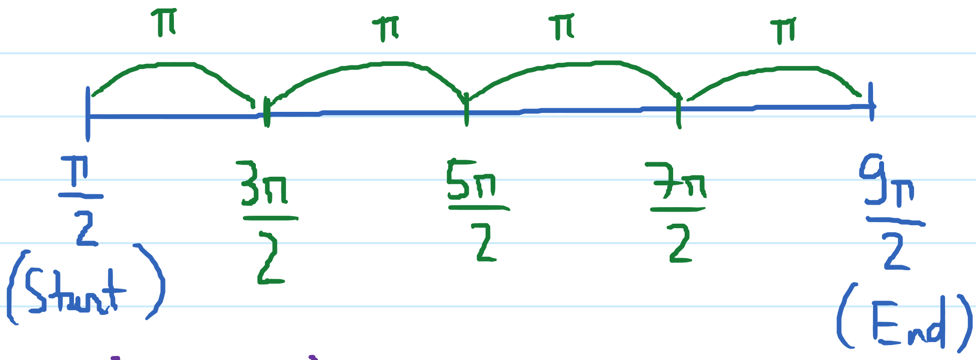
Graph of $y = a\sin(bx-c)+d$ or $y=\cos(bx-c)+d$ can be obtained from the graph of $y = a\sin(bx-c)$ or $y=\cos(bx-c)$ by shifting the latter up or $(d>0)$ down d units. $(d<0)$

E.g. Graph: $y = 1 + 2\sin\left(\boxed{\frac{1}{2}x - \frac{\pi}{4}}\right)$ in 1 period.

$$\frac{1}{2}x - \frac{\pi}{4} = 2\pi$$
$$\frac{1}{2}x = 2\pi + \frac{\pi}{4}$$
$$\frac{1}{2}x = \frac{9\pi}{4}$$
$$x = \frac{9\pi}{2}$$

Amplitude $= 2$; period $= 4\pi$.

Endpoints: $\frac{\pi}{2}$ (Start) ; $\frac{9\pi}{2}$ (End)



"Intercept"	Max	"Intercept"	Min	"Intercept"
$\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{3\pi}{2}, 3\right)$	$\left(\frac{5\pi}{2}, 1\right)$	$\left(\frac{7\pi}{2}, -1\right)$	$\left(\frac{9\pi}{2}, 1\right)$

$\longrightarrow$ graph.