4.3 and 4.4. Graphs of the Remaining Trig Functions.

Monday, March 4, 2019 10:27 AM

① The tangent function.

$$y = \tan x = \frac{\sin x}{\cos x}$$

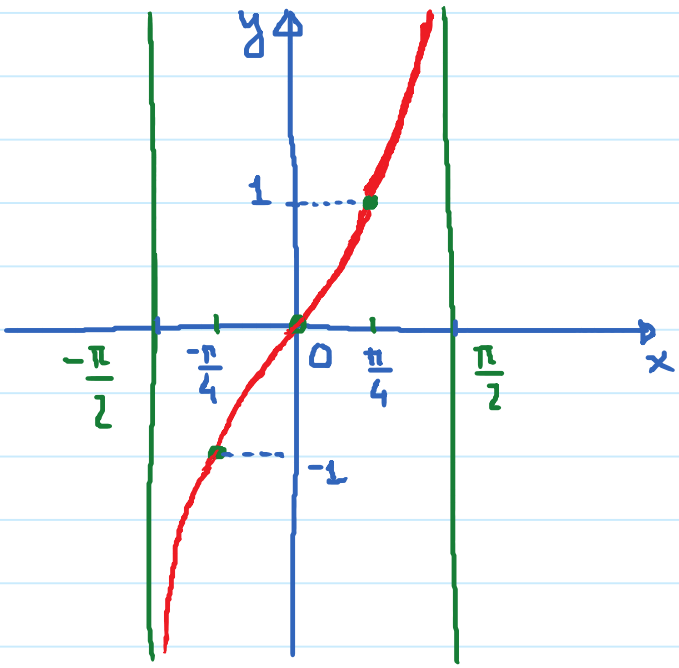
The tangent is undefined when $\cos x = 0$.

$\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, \ldots$

Period of tangent function is π.

x	$y = \tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined

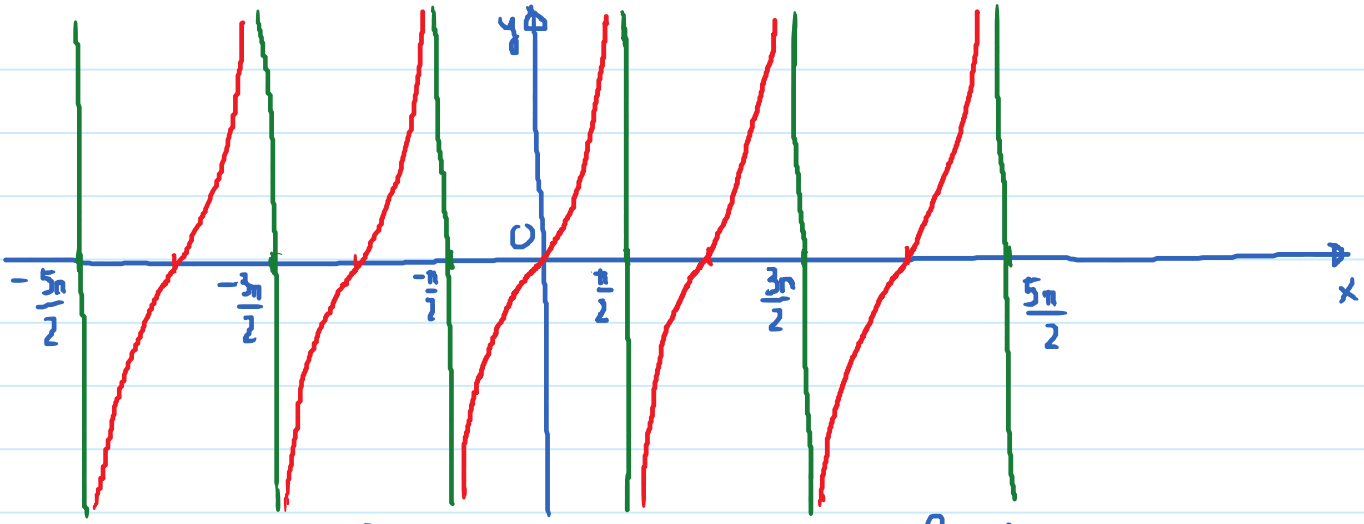


No amplitude.

Period: π.

Pattern: V.A. Down Intercept up V.A.

(Start) $\frac{1}{2}$ period (End)



Complete graph of tangent function.

* Process for graphing $y = a\tan(bx - c) + d$.

Find 1 cycle: Set $bx - c = -\frac{\pi}{2}$ and $bx - c = \frac{\pi}{2}$

↓ Solve to get left endpt

↓ solve to get right endpoint

V.A. "Intercept" V.A.

Start $\frac{1}{2}$ period End

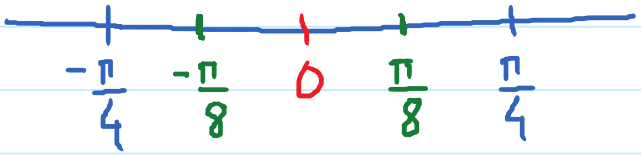
Period: $\frac{\pi}{b}$

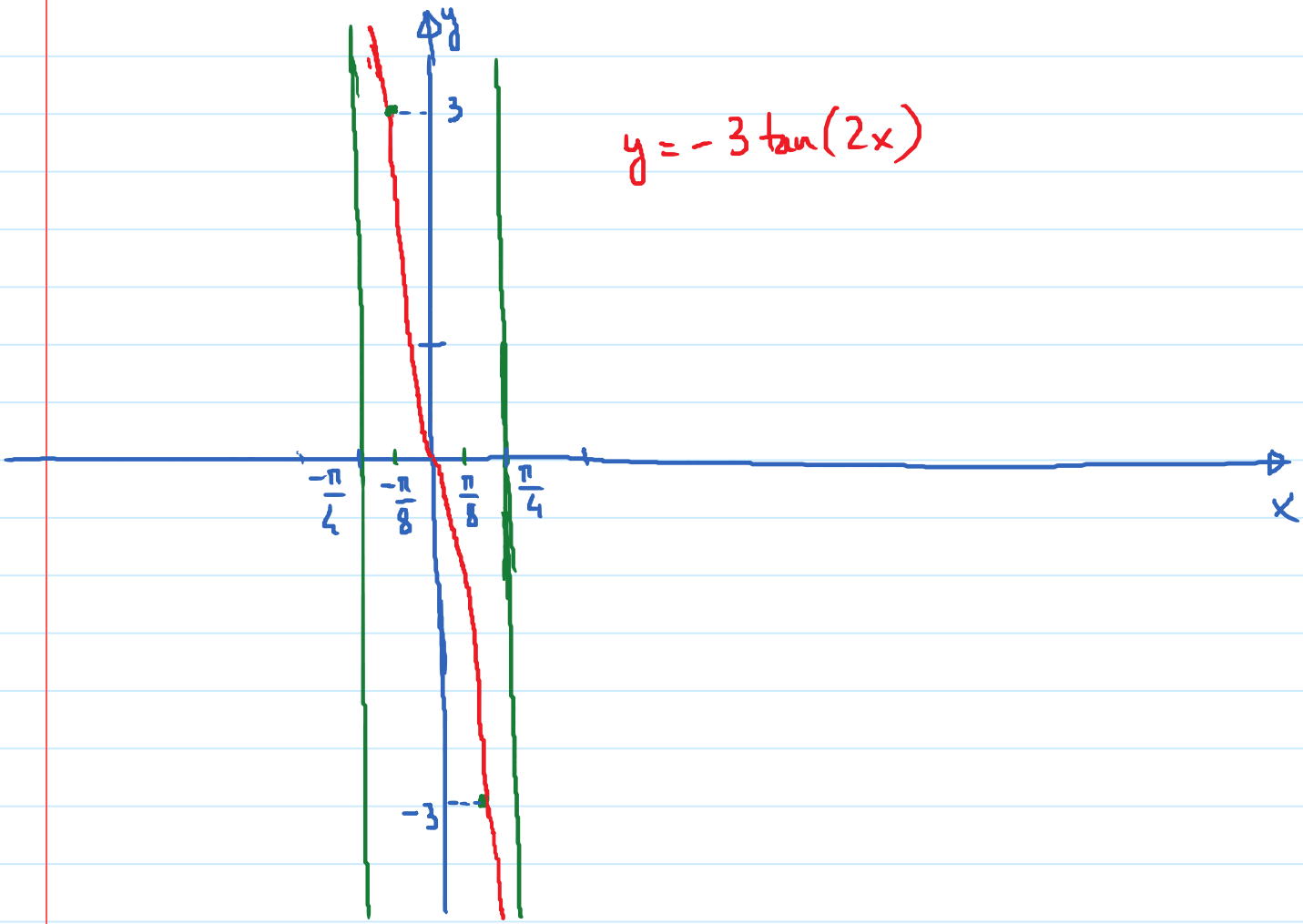
E.g. Graph $y = -3\tan(2x)$ in 1 period.

Period $= \frac{\pi}{2}$. $2x = -\frac{\pi}{2}$; $2x = \frac{\pi}{2}$

$x = -\frac{\pi}{4}$; $x = \frac{\pi}{4}$.

V.A 3 Intercept -3 V.A.





$y = -3\tan(2x)$

(2) The Cotangent function:

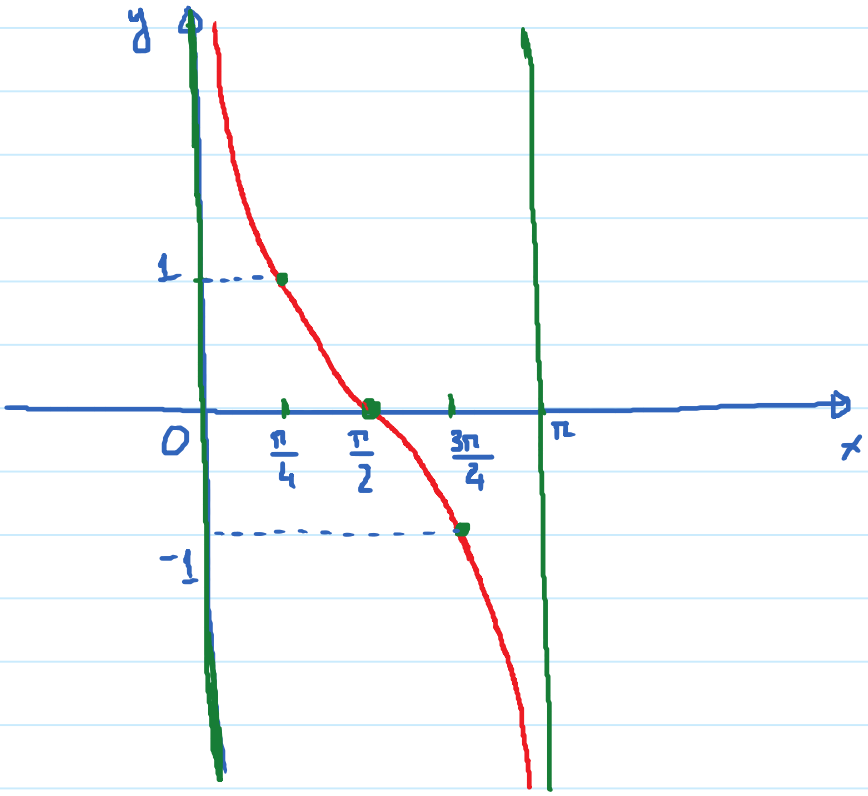
$$y = \cot x = \frac{\cos x}{\sin x}$$

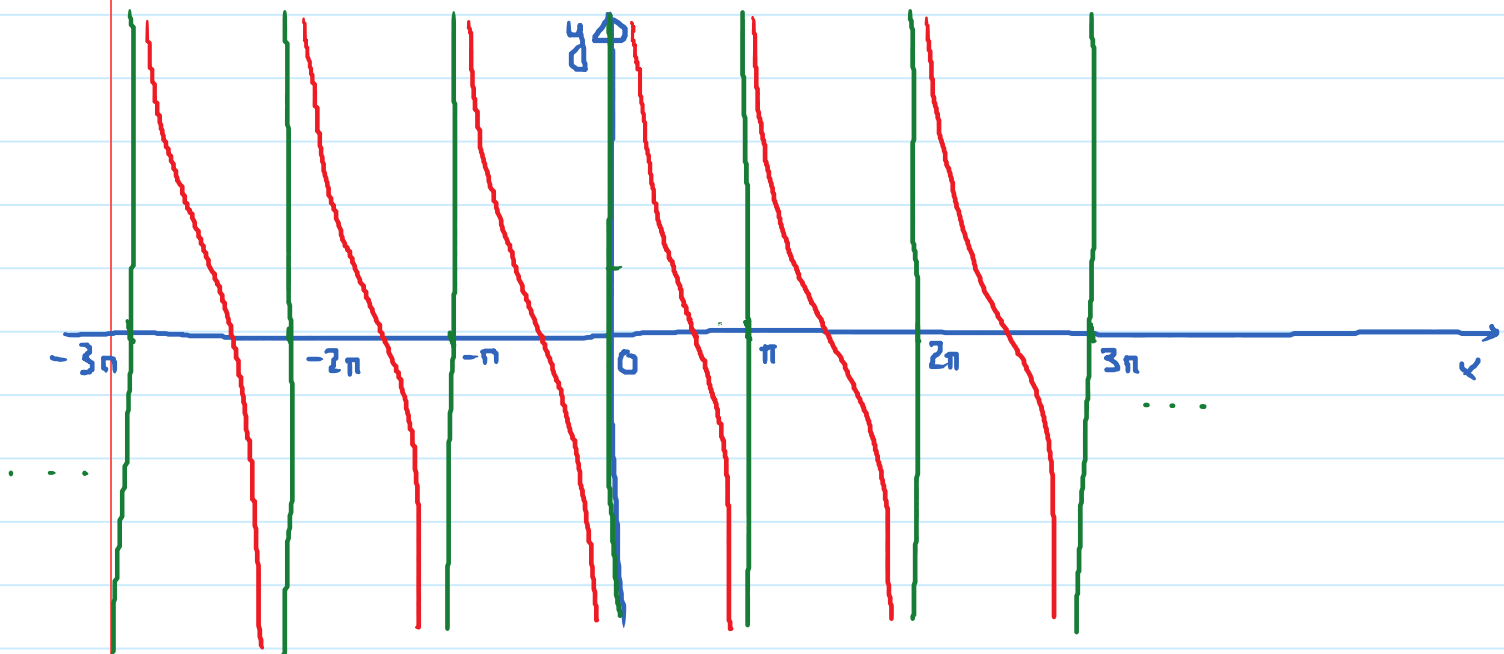
cotangent is undefined when $\sin x = 0$

$\sin x = 0$ when $x = 0, \pi, 2\pi, 3\pi, \ldots, -\pi, -2\pi, -3\pi, \ldots$

Period = π.

x	$y = \cot x$
0	undefined
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined





Complete graph of cotangent.

Process for graphing $y = a\cot(bx-c)+d$ in 1 period

Endpoints: $bx - c = 0 \rightarrow$ left V.A.

$bx - c = \pi \rightarrow$ right V.A.

Period : $\frac{\pi}{b}$.

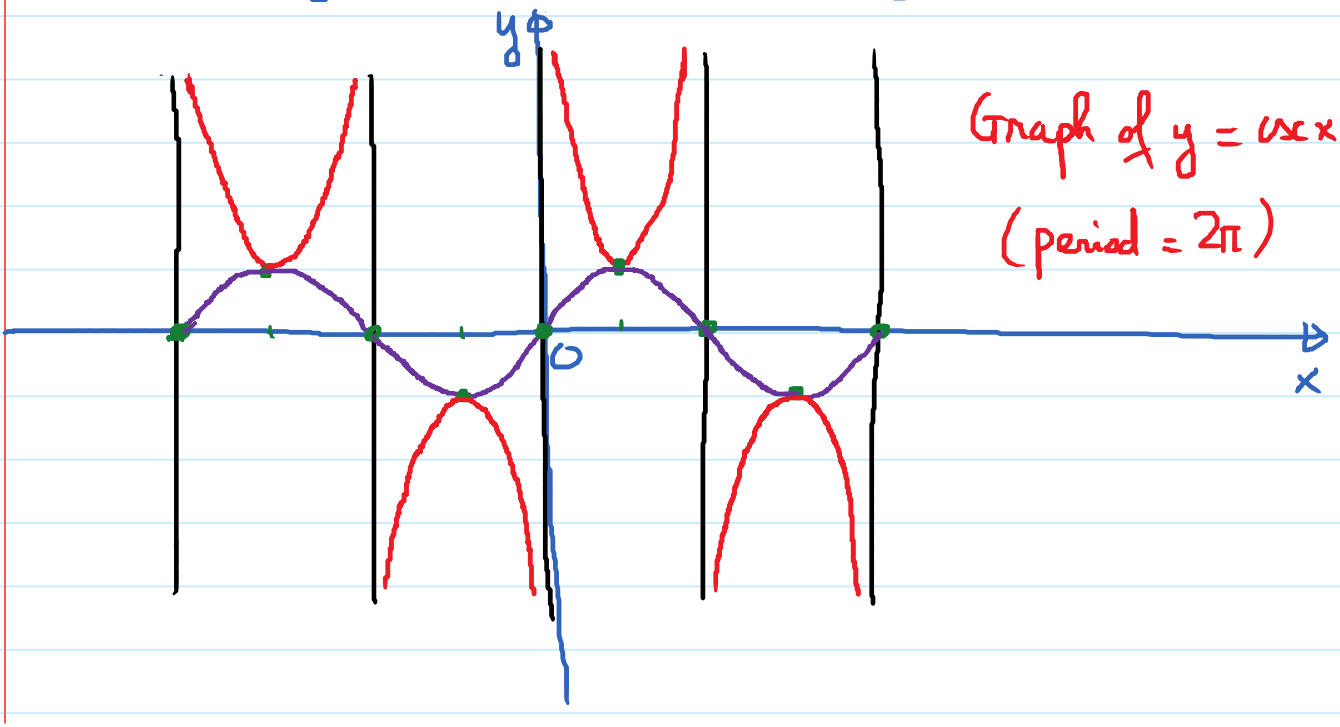
(3) Graphs of cosecant function and secant function.

$y = \csc x = \frac{1}{\sin x}$.

cosecant is undefined when $\sin x = 0$

$\sin x = 0$ when $x = 0, \pi, 2\pi, 3\pi, \ldots, -\pi, -2\pi, \ldots$

To graph $y = \csc x$, we start by graph $y = \sin x$



Graph of $y = \csc x$

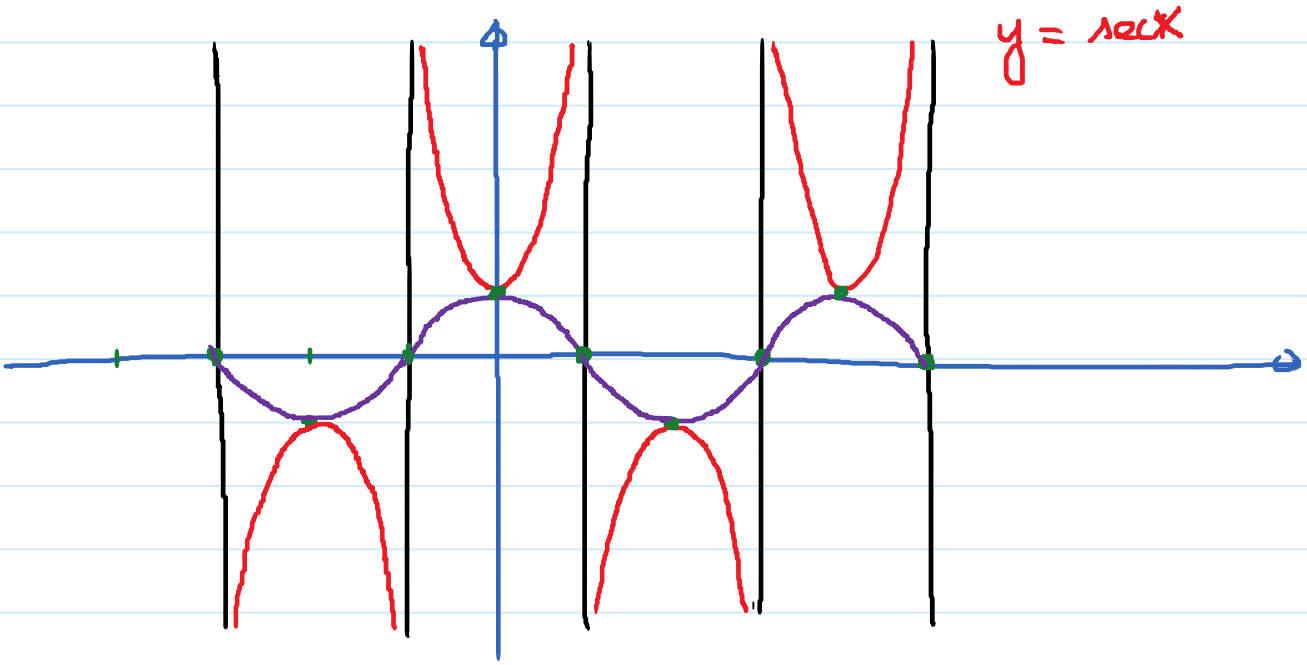
(period $= 2\pi$)

$$y = \sec x = \frac{1}{\cos x}$$

$\sec x$ is undefined when $\cos x = 0$; it is when

$$x = \ldots -\frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Sketch the graph of $y = \sec x$



* To graph $y = a\sec(bx-c)$ or $y = a\csc(bx-c)$

We just need to graph the corresponding sine and cosine function first and use them to locate asymptotes and other points of graph.

E.g. Graph $y = 2\csc\left(x + \frac{\pi}{4}\right)$

We start by graphing $y = 2\sin\left(x + \frac{\pi}{4}\right)$

Amplitude = 2 ; period : 2π.

Endpoints : $x + \frac{\pi}{4} = 0$ and $x + \frac{\pi}{4} = 2\pi$.

$x = -\frac{\pi}{4}$; $x = \frac{7\pi}{4}$.

Intercept	Max	Intercept	Min	Intercept
$\left(-\frac{\pi}{4}, 0\right)$	$\left(\frac{\pi}{4}, 2\right)$	$\left(\frac{3\pi}{4}, 0\right)$	$\left(\frac{5\pi}{4}, -2\right)$	$\left(\frac{7\pi}{4}, 0\right)$