5.4 Sum and Difference Identities for Sine and Tangent

Wednesday, March 27, 2019 10:30 AM

Recall:

$$\cos(A+B) = \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B)$$

$$\cos(A-B) = \cos(A)\cdot\cos(B) + \sin(A)\cdot\sin(B)$$

Sum / Difference Identities for Sine.

$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cdot\cos(A)$$

$$\sin(A-B) = \sin(A)\cdot\cos(B) - \sin(B)\cdot\cos(A)$$

E.g. $\sin(105°)$

$$\sin(60° + 45°) = \sin(60°)\cos(45°) + \sin(45°)\cdot\cos(60°)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}} \rightarrow \sin(105°)$$

E.g. $\sin(270° - \theta)$. Write this as a single function of θ.

$$\sin(270^\circ - \theta) = \sin(270^\circ)\cos(\theta) - \sin(\theta)\cdot\cos(270^\circ)$$

$$= (-1)\cdot\cos(\theta) - \sin(\theta)\cdot 0$$

$$= \boxed{-\cos(\theta)}$$

Sum/Difference Identities for tangent.

$$\boxed{\begin{aligned} \tan(A+B) &= \frac{\tan(A) + \tan(B)}{1 - \tan(A)\cdot\tan(B)} \\ \tan(A-B) &= \frac{\tan(A) - \tan(B)}{1 + \tan(A)\cdot\tan(B)}. \end{aligned}}$$

* Verify the first identity.

$$LHS = \tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\big(\sin(A)\cos(B) + \sin(B)\cos(A)\big) / \big(\cos(A)\cos B\big)}{\big(\cos(A)\cos(B) - \sin(A)\sin(B)\big) / \big(\cos(A)\cos(B)\big)}$$

$$= \frac{\frac{\sin(A)\cancel{\cos(B)}}{\cos(A)\cancel{\cos(B)}} + \frac{\sin(B)\cancel{\cos(A)}}{\cancel{\cos(A)}\cos(B)}}{\frac{\cos(A)\cos(B)}{\cos(A)\cos(B)} - \frac{\sin(A)\sin(B)}{\cos(A)\cos(B)}}$$

$$= \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)} = RHS.$$

E.g. Find the exact value of $\tan(15°)$

$$\tan(15°) = \tan(45° - 30°) = \frac{\tan(45°) - \tan(30°)}{1 + \tan(45°)\tan(30°)}$$

$$= \frac{\frac{3\cdot 1}{3\cdot 1} - \frac{\sqrt{3}}{3}}{\frac{3\cdot 1}{3\cdot 1} + 1\cdot\frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$

$$= \frac{3-\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}}$$

$$= \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{(3-\sqrt{3})\cdot(3-\sqrt{3})}{6} = \frac{9 - 6\sqrt{3} + 3}{6}$$

$$\frac{12 - 6\sqrt{3}}{6}$$

$$\frac{\cancel{6}\boxed{(2-\sqrt{3})}}{\cancel{6}}$$

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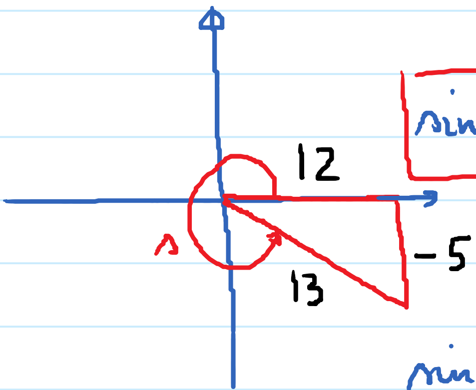
tangent of a sum

E.g. $\dfrac{\tan\frac{3\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)} = \tan\left(\underbrace{\frac{3\pi \cdot 3}{4 \cdot 3} + \frac{\pi \cdot 2}{6 \cdot 2}}\right)$

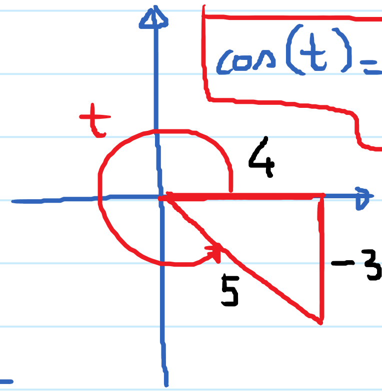
$= \boxed{\tan\left(\frac{11\pi}{12}\right)}$

E.g. $\cos(s) = \frac{12}{13}$; $\sin(t) = -\frac{3}{5}$; s and t are in Q 4

(a) $\sin(s+t) = \underbrace{\boxed{\sin(s)}\boxed{\cos(t)}}_{\text{missing}} + \underbrace{\boxed{\sin(t)}}_{\text{given}}\underbrace{\boxed{\cos(s)}}_{\text{given}}$



$\boxed{\sin(s) = -\frac{5}{13}}$



$\boxed{\cos(t) = \frac{4}{5}}$

$\sin(s+t) = -\frac{56}{65}$

(b)

$$\tan(s+t) = \frac{\tan(s) + \tan(t)}{1 - \tan(s)\cdot\tan(t)}$$

$\tan(s) = \dfrac{\sin(s)}{\cos(s)} = \dfrac{-5/13}{12/13} = -\dfrac{5}{12}$; $\tan(t) = \dfrac{\sin(t)}{\cos(t)} = -\dfrac{3}{4}$

$$\tan(s+t) = \frac{-\frac{5}{12} + \left(-\frac{3}{4}\right)}{1 - \left(-\frac{5}{12}\right)\cdot\left(-\frac{3}{4}\right)} = -\frac{56}{33}$$

(c) Which quadrant is $(s+t)$ in?

$\sin(s+t) < 0$; $\tan(s+t) < 0$

$\longrightarrow s+t$ is in QIV

E.g.

$$\tan(60^\circ - x) = \frac{\tan(60^\circ) - \tan(x)}{1 + \tan(60^\circ)\tan(x)}$$

$$= \frac{\sqrt{3} - \tan(x)}{1 + \sqrt{3}\tan(x)}$$