5.5. Double Angle Identities

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Double Angle Identities for cosine:

$$\boxed{\begin{aligned} \cos(2A) &= 2\cos^2(A) - 1. & \text{I} \\ \cos(2A) &= 1 - 2\sin^2(A) & \text{II} \\ \cos(2A) &= \cos^2(A) - \sin^2(A) & \text{III} \end{aligned}}$$

Verify the identity

III $\quad \cos(2A) = \cos^2(A) - \sin^2(A)$

LHS $= \cos(2A) = \cos(A+A)$

$= \cos(A)\cdot\cos(A) - \sin(A)\cdot\sin(A)$

cosine of sum

$= \cos^2(A) - \sin^2(A) = \text{RHS}.$

* I and II are direct consequences of III

Verify I

$\cos(2A) = \cos^2(A) - \sin^2(A)$

$= \cos^2(A) - [1 - \cos^2(A)]$

$= \cos^2(A) - 1 + \cos^2(A) = 2\cos^2(A) - 1$

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$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$= 1 - \sin^2(A) - \sin^2(A)$$

$$= 1 - 2\sin^2(A) \rightarrow \text{Verify II}$$

Double Identities for Sine.

$$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$$

Verify:

$$LHS = \sin(2A) = \sin(A + A)$$

$$= \sin(A)\cos(A) + \cos(A)\cdot\sin(A)$$

sine of a sum

$$= 2\sin(A)\cos(A) = RHS.$$

Double angle Identity for tangent.

$$\boxed{\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}}$$

$$LHS = \tan(2A) = \tan(A + A) = \frac{\tan(A) + \tan(A)}{1 - \tan(A)\cdot\tan(A)}$$

$$= \frac{2\tan(A)}{1 - \tan^2(A)} = RHS.$$

Summary: All Double angle Identities:

$$\boxed{\begin{aligned} \cos(2A) &= \begin{cases} \cos^2(A) - \sin^2(A) \\ 2\cos^2(A) - 1 \\ 1 - 2\sin^2(A) \end{cases} \\ \sin(2A) &= 2\sin(A)\cos(A) \\ \tan(2A) &= \frac{2\tan(A)}{1 - \tan^2(A)} \end{aligned}}$$

E.g. Given: $\cos(\theta) = -\frac{12}{13}$ and $\sin(\theta) > 0$

Find $\cos(2\theta)$. Find $\sin(2\theta)$

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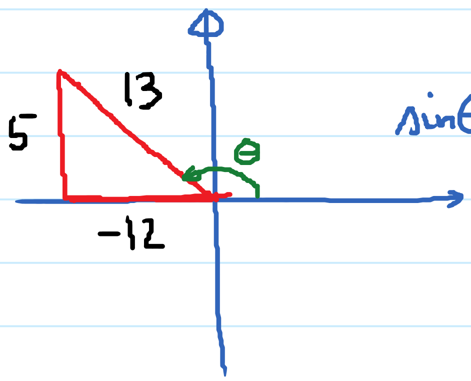
* $\cos(2\theta) = 2\cos^2(\theta) - 1$

$$= 2\cdot\left(-\frac{12}{13}\right)^2 - 1 = \boxed{\frac{119}{169}}$$

* $\sin(2\theta) = 2\underbrace{\boxed{\sin(\theta)}}_{\text{missing}}\underbrace{\boxed{\cos(\theta)}}_{\text{given}}$



$\sin\theta = \frac{5}{13}$

$$\sin(2\theta) = 2\cdot\frac{5}{13}\cdot\left(-\frac{12}{13}\right) = \boxed{-\frac{120}{169}}$$

E.g. Given: $\cos(2\theta) = -\frac{28}{53}$ and θ is in QII.

Find $\sin(\theta)$, Find $\cos(\theta)$.

* $\cos(2\theta) = 1 - 2\sin^2(\theta)$

$\cos(2\theta) - 1 = -2\sin^2(\theta)$

$$\sin^2(\theta) = \frac{\cos(2\theta) - 1}{-2}$$

$$\sin(\theta) = \pm\sqrt{\frac{\cos(2\theta) - 1}{-2}}$$

Since θ is in QII, we have:

$$\sin(\theta) = \sqrt{\frac{-\frac{28}{53} - 1}{-2}} = \dots$$

* Find $\cos(\theta)$

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

$$\cos(2\theta) + 1 = 2\cos^2(\theta)$$

$$\cos^2(\theta) = \frac{\cos(2\theta) + 1}{2}$$

$$\cos(\theta) = \pm\sqrt{\frac{\cos(2\theta) + 1}{2}}$$

θ is in QII, so, $\cos\theta = -\sqrt{\frac{\cos(2\theta)+1}{2}}$

$$\cos\theta = -\sqrt{\frac{-\frac{28}{53} + 1}{2}} = \dots$$