5.6. Half-Angle Identities.

Monday, April 1, 2019 10:47 AM

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}} \; ; \; \sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}$$

The + or − depends on which quadrant the angle $\frac{A}{2}$ belongs to.

$$\tan\left(\frac{A}{2}\right) = \begin{cases} \pm\sqrt{\dfrac{1-\cos(A)}{1+\cos(A)}} \\ \dfrac{\sin(A)}{1+\cos(A)} \\ \dfrac{1-\cos(A)}{\sin(A)} \end{cases}$$

E.g. Use the half angle identities to find the exact value of

(a) $\sin(15^\circ)$ (b) $\tan(195^\circ)$

E.g. Given $\cos\theta = -\frac{5}{8}$ and $\frac{\pi}{2} < \theta < \pi$.

$90^\circ < \theta < 180^\circ$

Find $\sin\left(\frac{\theta}{2}\right)$ $\rightarrow 45^\circ < \frac{\theta}{2} < 90^\circ$

E.g. Given $\tan\theta = \frac{\sqrt{7}}{3}$ and $180^\circ < \theta < 270^\circ$.

Find $\tan\left(\frac{\theta}{2}\right)$.

Sol:

$$\sin(15^\circ) = \sqrt{\frac{1-\cos(30^\circ)}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} \rightarrow \sin 15^\circ$$

$$\sin(360^\circ + 30^\circ)$$

* $$\tan(195^\circ) = \frac{\boxed{\sin(390^\circ)}}{1 + \cos(390^\circ)} \rightarrow \cos(360^\circ + 30^\circ)$$

$$= \frac{\sin(30^\circ)}{1 + \cos(30^\circ)}$$

$$= \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$$

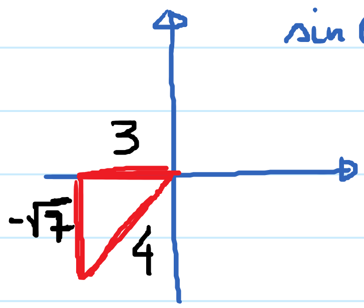
$$\tan(195^\circ) = \boxed{\frac{1}{2+\sqrt{3}}}$$

* $$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$45^\circ < \frac{\theta}{2} < 90^\circ \rightarrow \frac{\theta}{2}$ is in QI $\rightarrow$ choose +

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \left(-\frac{5}{8}\right)}{2}} = \ldots$$

* $$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$? ?



$$\sin\theta = \frac{-\sqrt{7}}{4}$$

$$\cos\theta = \frac{3}{4}.$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{-\frac{\sqrt{7}}{4}}{1 + \frac{3}{4}} = \frac{-\frac{\sqrt{7}}{4}}{\frac{7}{4}} = -\frac{\sqrt{7}}{7}$$